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THE APRIL MEETING IN NEW YORK

The three hundred twenty-second meeting of the American Mathematical Society was held at Columbia University on Friday and Saturday, April 19–20, 1935. The attendance included the following one hundred fifty-two members of the Society:

C. R. Adams, R. P. Agnew, E. B. Allen, R. C. Archibald, N. H. Ball, J. L. Barnes, M. F. Becker, Garrett Birkhoff, G. D. Birkhoff, H. L. Black, H. F. Blichfeldt, Salomon Bochner, H. F. Bohnenblust, J. W. Bower, C. B. Boyer, Richard Brauer, H. W. Brinkmann, A. B. Brown, E. W. Brown, S. S. Cairns, Alonzo Church, W. R. Church, J. A. Clarkson, W. S. Claytor, George Comenetz, Byron Cosby, II, Richard Courant, E. H. Cutler, J. L. Doob, H. L. Dorwart, Jesse Douglas, F. G. Dressel, L. P. Eisenhart, W. W. Elliott, T. S. Fiske, W. B. Fite, M. M. Flood, R. M. Foster, J. S. Frame, Orrin Frink, T. C. Fry, C. A. Garabedian, H. M. Gehman, B. P. Gill, D. C. Gillespie, W. O. Gordon, P. H. Graham, M. C. Gray, J. A. Greenwood, C. C. Grove, N. L. Haight, Marshall Hall, G. A. Hedlund, M. R. Hestenes, E. H. C. Hildebrandt, Einar Hille, Lulu Hofmann, Charles Hopkins, G. M. Hopper, Harold Hotelling, E. M. Hull, W. H. Ingram, Nathan Jacobson, S. A. Joffe, R. A. Johnson, F. E. Johnston, I. N. Kagno, E. R. van Kampen, Edward Kasner, L. S. Kennison, S. C. Kleene, J. R. Kline, H. L. Krall, Harry Langman, V. V. Latshaw, Solomon Lefschetz, Jack Levine, D. C. Lewis, Jr., Hans Lewy, I. C. McLaughlin, E. J. McShane, L. A. MacColl, Saunders MacLane, N. B. MacLean, H. M. MacNeille, H. F. MacNeish, R. J. Marcou, W. T. Martin, A. K. Mitchell, C. N. Moore, T. W. Moore, G. W. Mullins, S. B. Myers, D. S. Nathan, E. P. Northrop, C. O. Oakley, Alta Odoms, F. W. Owens, H. B. Owens, H. B. Phillips, R. S. Pieters, G. B. Price, R. G. Putnam, H. A. Rademacher, J. F. Randolph, H. W. Raudenbush, Jr., G. E. Raynor, H. W. Reddick, M. S. Rees, R. G. D. Richardson, J. F. Ritt, J. H. Roberts, L. B. Robinson, S. L. Robinson, J. B. Rosenbach, J. B. Rosser, S. G. Roth, O. K. Sagen, M. F. Schmeiser, I. J. Schoenberg, I. M. Sheffer, Max Shiffman, C. G. Shover, L. G. Simons, M. E. Sinclair, Abraham Sinkov, A. A. Stafford, J. D. Tamarkin, J. M. Thomas, J. E. Thompson, J. L. Vanderslice, Oswald Veblen, Henry Wallman, J. L. Walsh, R. M. Walter, J. F. Wardwell, G. C. Webber, M. S. Webster, H. S. White, E. A. Whitman, Hassler Whitney, D. V. Widder, Norbert Wiener, S. S. Wilks, A. H. Wilson, Aurel Wintner, H. P. Wirth, D. W. Woodard, E. W. Woolard, Oscar Zariski, Leo Zippin, Max Zorn.

The meeting opened on Friday morning with two sectional sessions, Algebra and Number Theory, and Geometry and Foundations. Friday afternoon was devoted to a Symposium on *The problem of three bodies and allied problems*, at which the following addresses were presented: *The present status of the restricted problem of three bodies*, by Professor G. D. Birkhoff;

Problems suggested by the general theories of planetary and satellite motions, by Professor E. W. Brown; *Periodic lunar orbits and gaps of commensurability*, by Dr. Aurel Wintner. A general session was held on Saturday morning.

At the opening of the session on Friday afternoon President Lefschetz spoke of the death of Professor Emmy Noether on Sunday, April 14th, at Bryn Mawr. The Society stood in silence for a minute in honor of the memory of this great mathematician.

A meeting of the Council was held at 5:00 P.M. on Friday in the Faculty Club of Columbia University. The election of the following persons to membership in the Society was announced by the Secretary:

Professor Mae Ruth Anderson, Concordia College, Moorhead, Minn.;
Dr. William Martin Borgman, Jr., Wayne University;
Dr. Foster Lindsey Brooks, Carrollton High School, Carrollton, Ohio;
Dr. Byron Cosby, II, Rensselaer Polytechnic Institute;
Mr. Carleton Craig, McGill University;
Dr. George Graham Harvey, Massachusetts Institute of Technology;
Mr. Emerson D. Jenkins, Columbus, Ohio;
Dr. Gertrude Stith (Mrs. P. W.) Ketchum, Urbana, Ill.;
Professor John Franklin Locke, State Teachers College, Memphis, Tenn.;
Mr. John Charles Chenoweth McKinsey, University of California;
Professor René Joseph Marcou, Boston College;
Sister Marie Gertrude McNeil, Seton Hill College, Greensburg, Pa.;
Dr. William Ted Martin, Princeton University;
Sister Mary Cleophas Garvin, Notre Dame College, South Euclid, Ohio;
Mr. Arthur Everett Pitcher, Harvard University;
Dr. Oswald Karl Sagen, Institute for Advanced Study;
Mr. E. Dillon Smith, Columbia University;
Professor James Johnston Stoker, Jr., Carnegie Institute of Technology;
Professor William Timothy Stratton, Kansas State College;
Sister Thomas Marie Maloney, Trinity College, Washington, D.C.;
Professor Julian Leroy Thompson, Emmanuel Missionary College, Berrien Springs, Mich.;
Professor Blanche Hall Tolar, Fenn College, Cleveland, Ohio;
Mr. Thirukkannapuram Vijayaraghavan, University of Dacca;
Professor Thomas Leonard Wade, Mercer University;
Mr. Henry Wallman, Princeton University;
Mr. Everett Thomas Welmers, Ann Arbor, Mich.

As nominee of the Members of the Department of Mathematics at the College of the City of New York:
Mr. Max Shiffman, Long Island, N. Y.

As nominee of the Massachusetts Institute of Technology:
Mr. Arra Steve Avakian, Massachusetts Institute of Technology.

As nominees of the University of Pennsylvania:
Dr. Harold M. Lufkin, University of Pennsylvania;
Mr. Robert Reed Lyle, University of Pennsylvania;
Dr. Adam J. Smith, University of Pennsylvania;
Miss Vivian Eberle Spencer, University of Pennsylvania.

As nominees of Wesleyan University:
Mr. Frank Hackett Byron, Wesleyan University;
Mr. Winthrop Thatcher Lewis, Harvard University.

As nominees of the Department of Mathematics of Yale University:
Mr. Melvin Dresher, Yale University;
Miss Jeanette Fox, New Haven, Conn.

As nominees of Indiana University:
Mr. Walter George Gingery, George Washington High School, Indianapolis, Ind.;
Dr. Sidney Guy Hacker, Indiana University.

In accordance with a reciprocity agreement:
Professor K. S. K. Iyengar, Central College, Bangalore, India.

The following twelve institutions of learning and mathematical groups were elected to institutional membership in the Society:

California Institute of Technology;
Carnegie Institute of Technology;
Catholic University of America;
Haverford College;
Indiana University;
Iowa State College;
State University of Iowa;
University of North Carolina;
Library of Purdue University;
Departments of Mathematics and Applied Mathematics of the University of Toronto;
Department of Mathematics of Williams College;
Department of Mathematics of the University of New Mexico.

It was reported that Professor H. L. Smith had been appointed as the representative of the Society at the Conference on Education in connection with the celebration of the seventy-fifth anniversary of the founding of the Louisiana State University, April 9–12, 1935.

It was announced that Mr. Thirukkannapuram Vijayaraghavan of the University of Dacca, India, had accepted the invitation to be the Society Visiting Lecturer for the year 1936.

The time of the October, 1935, meeting in New York was set for October 26. Announcement was made that the invitation from the George Washington University to hold the Annual Meeting of 1936 at that institution in connection with the meetings of the American Association for the Advancement of Science had been accepted.

As Committee on Arrangements for the 1936 Summer Meeting to be held at Harvard University in connection with the Tercentenary Celebration, the Council appointed Professor G. D. Birkhoff (chairman), Mrs. M. C. Graustein, Professors H. B. Phillips, R. G. D. Richardson, and M. H. Stone.

Professor C. N. Moore was appointed as the representative of the Society on the Editorial Board of the American Yearbook for a period of three years beginning with 1935.

The conditions for eligibility for the Society's prizes were modified so as to include memoirs published in any journal printed in the United States or Canada.

Titles and cross references to the abstracts of the papers (other than symposium addresses) read at this meeting follow below; papers whose abstract numbers are followed by the letter *t* were read by title. The papers numbered 1 to 11 were read before the section of Algebra and Number Theory, Professor J. F. Ritt presiding; those numbered 12 to 22 before the section of Geometry and Foundations, Professors D. C. Gillespie and J. D. Tamarkin presiding; those numbered 23 to 63 at the general session, Professors Norbert Wiener and Edward Kasner presiding. Professor Solomon Lefschetz presided at the Symposium. Mr. Dunford was introduced by Professor Tamarkin, Mr. Fox by Professor Kline, Mr. Hill by Professor Adams, Professor Mayer by Professor Tamarkin, Dr. Quine by Mr. Birkhoff, Dr. MacGregor by Professor Kline, Professor Kogbetliantz by Professor Tamarkin.

1. *Generalization of Lucas' u_n, v_n for recurrences of third and higher order* (preliminary report), by Professor B. P. Gill. (Abstract No. 41-5-184.)

2. *On a certain function connected with polynomials in a Galois field*, by Professor Leonard Carlitz. (Abstract No. 41-5-185-*t*.)

3. *The automorphisms of Cayley's non-associative algebra* (preliminary report), by Dr. Max Zorn. (Abstract No. 41-5-186.)

4. *Indices in cubic fields*, by Mr. Marshall Hall. (Abstract No. 41-5-187.)

5. *An unsolvable problem of elementary number theory* (preliminary report), by Professor Alonzo Church. (Abstract No. 41-5-205.)

6. *Concerning transcendental numbers*, by Dr. G. C. Webber (National Research Fellow). (Abstract No. 41-5-188.)

7. *The groups determined by the relations* $S^l = T^m = (S^{-1}T^{-1}ST)^p = 1$, by Dr. H. S. M. Coxeter and Dr. Abraham Sinkov. (Abstract No. 41-5-189.)

8. *Locally compact totally disconnected rings*, by Dr. Nathan Jacobson. (Abstract No. 41-5-190.)

9. *Theory of finite distributive free structures*, by Mr. W. R. Church. (Abstract No. 41-5-191.)

10. *Continuous transformations preserving all topological properties*, by Mr. J. F. Wardwell. (Abstract No. 41-5-253.)

11. *Homogeneous sets which are disconnected by the removal of a finite number of points*, by Dr. S. L. Robinson. (Abstract No. 41-5-220.)

12. *Postulate systems for Lie groups*, by Mr. Garrett Birkhoff. (Abstract No. 41-5-192.)

13. *Some density properties of point sets*, by Dr. J. F. Randolph. (Abstract No. 41-3-176.)

14. *Extensions of partially ordered sets* (preliminary report), by Mr. H. M. MacNeille. (Abstract No. 41-3-142.)

15. *The duals of velocity families and natural families*, by Professor Edward Kasner. (Abstract No. 41-5-193.)

16. *Connections between differential geometry and topology.* II: *Closed surfaces*, by Dr. S. B. Myers (National Research Fellow). (Abstract No. 41-5-194.)

17. *Systems of curves and surfaces related under cyclic involutions*, by Professor H. L. Black. (Abstract No. 41-5-195.)

18. *Collections filling a plane*, by Professor J. H. Roberts. (Abstract No. 41-5-196.)

19. *The formal theory of conservative transformations in 2n-dimensional space*, by Dr. D. C. Lewis (National Research Fellow). (Abstract No. 41-5-197.)

20. *Irreducible non-toral graphs*, by Mr. I. N. Kagno. (Abstract No. 41-5-198.)

21. *Sphere-spaces, with applications*, by Professor Hassler Whitney. (Abstract No. 41-5-222.)

22. *A topological proof of the Riemann-Roch theorem*, by Professor Oscar Zariski. (Abstract No. 41-5-252.)

23. *Integration in general analysis*, by Mr. Nelson Dunford. (Abstract No. 41-5-199.)

24. *Natural isoperimetric conditions for variable end point problems in the calculus of variations*, by Professor G. D. Birkhoff and Dr. M. R. Hestenes. (Abstract No. 41-3-173.)

25. *On the multiplication of series summable by Nörlund means*, by Professor C. N. Moore. (Abstract No. 41-5-200.)

26. *Summation of double Fourier series by circles*, by Professor Salomon Bochner. (Abstract No. 41-5-201.)

27. *Distribution functions and the Riemann zeta function*, by Dr. Borge Jessen and Dr. Aurel Wintner. (Abstract No. 41-3-174.)

28. *A differential equation for Appell polynomials*, by Professor I. M. Sheffer. (Abstract No. 41-5-202.)

29. *The zeros of Jacobi polynomials*, by Mr. M. S. Webster. (Abstract No. 41-3-139.)

30. *Abstract absolute values and polygonal irreducibility criteria*, by Dr. Saunders MacLane. (Abstract No. 41-5-221.)

31. *Concerning an intrinsic property of plane continua*, by Dr. R. E. Basye. (Abstract No. 41-3-171-*t*.)

32. *On subharmonic functions*, by Dr. E. F. Beckenbach. (Abstract No. 41-3-183-*t*.)

33. *On the lattice theory of linear dependence*, by Mr. Garrett Birkhoff. (Abstract No. 41-5-203-*t*.)

34. *Oscillation, separation, and comparison theorems in the calculus of variations*, by Professor G. D. Birkhoff and Dr. M. R. Hestenes. (Abstract No. 41-3-143-*t*.)

35. *Boundary value problems and the calculus of variations*, by Professor G. D. Birkhoff and Dr. M. R. Hestenes. (Abstract No. 41-3-180-*t*.)

36. *Concerning metric transforms of metric spaces*, by Dr. L. M. Blumenthal (National Research Fellow). (Abstract No. 41-3-175-*t*.)

37. *Some properties of conversion*, by Professor Alonzo Church and Dr. J. B. Rosser. (Abstract No. 41-5-204-*t*.)

38. *Green's theorem for generalized harmonic functionals*, by Dr. F. G. Dressel. (Abstract No. 41-5-206-*t*.)

39. *On a theorem of Plessner*, by Mr. Nelson Dunford. (Abstract No. 41-5-207-*t*.)

40. *Differential equations with continuous spectra*, by Mr. A. H. Fox. (Abstract No. 41-5-208-*t*.)

41. *Prime divisors of second order sequences*, by Mr. Marshall Hall. (Abstract No. 41-5-209-*t*.)

42. *A metrically transitive group defined by the modular group*, by Professor G. A. Hedlund. (Abstract No. 41-5-210-*t*.)

43. *Some theorems on double limits*, by Mr. J. D. Hill. (Abstract No. 41-5-211-*t*.)

44. *Inter-relations among the four principal types of order*, by Professor E. V. Huntington. (Abstract No. 41-5-212-*t*.)

45. *The mathematical structure of Lewis's theory of strict implication*, by Professor E. V. Huntington. (Abstract No. 41-5-213-*t*.)

46. *The inversion of approximate derivatives*, by Professor R. L. Jeffery. (Abstract No. 41-5-214-*t*.)

47. *Characterization of the equilong group and a larger group*, by Professor Edward Kasner. (Abstract No. 41-5-215-*t*.)

48. *On distributions admitting a sufficient statistic*, by Professor B. O. Koopman. (Abstract No. 41-5-216-*t*.)

49. *A note on the multiple correlation coefficient*, by Dr. Solomon Kullback. (Abstract No. 41-3-141-*t*.)

50. *On the distribution problem of statistics*, by Dr. Solomon Kullback. (Abstract No. 41-3-170-*t*.)

51. *Functions of self-adjoint transformations in Hilbert space*, by Dr. E. R. Lorch. (Abstract No. 41-5-217-*t*.)

52. *The differential geometry of the submanifolds of the R_n of constant curvature*, by Professor Walther Mayer. (Abstract No. 41-3-182-*t*.)

53. *The relation of the classical orthogonal polynomials to the polynomials of Appell*, by Professor J. A. Shohat. (Abstract No. 41-5-218-*t*.)

54. *On criteria for rejection of observations and the distribution of the ratio of deviation to sample standard deviation*, by Dr. W. R. Thompson. (Abstract No. 41-3-172-*t*.)

55. *Concerning sequences and limiting sets*, by Professor G. T. Whyburn. (Abstract No. 41-5-219-*t*.)

56. *On free subsets of E_n*, by Professor R. L. Wilder. (Abstract No. 41-3-181-*t*.)

57. *An application of Laguerre polynomials*, by Professor D. V. Widder. (Abstract No. 41-5-224-*t*.)

58. *Postulates for effective equality and effective implication in formal logic*, by Professor E. V. Huntington. (Abstract No. 41-5-227-*t*.)

59. *Sets of independent axioms for complete Moore space and complete metric space*, by Dr. C. W. Vickery. (Abstract No. 41-5-226-*t*.)

60. *A unified calculus of propositions, classes, and relations*, by Dr. W. V. Quine. (Abstract No. 41-5-223-*t*.)

61. *Notes on linear transformations*. I, by Professor Einar Hille. (Abstract No. 41-5-250-*t*.)

62. *The potential function method for the solution of two-dimensional stress problems*, by Dr. C. W. MacGregor. (Abstract No. 41-5-225-*t*.)

63. *Linear functionals in the theory of quasi-analytic and D-analytic classes*, by Professor W. J. Trjitzinsky. (Abstract No. 41-5-251-*t*.)

64. *On the jump of a function determined by its Hermite or Laguerre series*, by Professor Ervand Kogbetliantz. (Abstract No. 41-5-254-*t*.)

J. R. KLINE,
Associate Secretary

THE APRIL MEETING IN CHICAGO

The three hundred twenty-third meeting of the American Mathematical Society was held at the University of Chicago on Friday and Saturday, April 19–20, 1935. There were about one hundred twenty-five persons in attendance, among whom were the following eighty-nine members of the Society:

A. A. Albert, W. L. Ayres, R. H. Bardell, R. W. Barnard, I. A. Barnett, Ralph Beatley, G. A. Bliss, W. M. Borgman, H. R. Brahana, R. W. Brink, Leonard Bristow, F. L. Brooks, L. E. Bush, W. H. Bussey, G. A. L. Caldwell, E. W. Chittenden, Max Coral, A. R. Crathorne, D. R. Curtiss, Wayne Dancer, L. E. Dickson, L. L. Dines, E. B. Escott, H. S. Everett, C. H. Fischer, L. M. Graves, L. W. Griffiths, E. H. Hanson, W. L. Hart, M. L. Hartung, T. H. Hildebrandt, J. J. L. Hinrichsen, D. L. Holl, Ralph Hull, M. H. Ingraham, Dunham Jackson, E. D. Jenkins, H. F. S. Jonah, W. C. Krathwohl, A. C. Ladner, E. P. Lane, R. E. Langer, C. G. Latimer, F. E. LeStourgeon, M. I. Logsdon, C. C. MacDuffee, W. D. MacMillan, M. L. MacQueen, Wilhelm Maier, H. W. March, Morris Marden, William Markowitz, Sister Mary de Lellis, A. E. May, J. R. Mayor, E. W. Miller, E. J. Moulton, A. L. Nelson, Rufus Oldenburger, G. A. Parkinson, H. P. Pettit, E. C. Pixley, H. H. Pixley, Tibor Radó, G. Y. Rainich, S. E. Rasor, W. T. Reid, R. F. Rinehart, A. E. Ross, W. E. Roth, N. E. Rutt, R. G. Sanger, Henry Scheffé, Henry Schultz, H. A. Simmons, Burke Smith, F. C. Smith, E. S. Sokolnikoff, I. S. Sokolnikoff, R. C. Stephens, H. P. Thielman, C. B. Tompkins, W. J. Trjitzinsky, J. I. Vass, H. E. Vaughan, H. S. Wall, R. L. Wilder, F. B. Wiley, F. E. Wood.

The meeting opened on Friday morning with two sectional sessions, one on Algebra and one on Analysis and Geometry. Friday afternoon Professor L. M. Graves of the University of Chicago gave a lecture entitled *Topics in functional calculus*. Saturday morning was devoted to a general session. During the first fifteen minutes Associate Secretary Ingraham reported on the progress of the campaign for the stabilization of the Society's finances.

Friday evening a dinner was held at Judson Court. About ninety mathematicians and their guests attended. Professor Tibor Radó of Ohio State University acted as toastmaster. He called first upon Professor E. J. Moulton of Northwestern University who discussed the employment situation among doctors in mathematics. Professor T. H. Hildebrandt of the University of Michigan next extended to the members an invitation to Ann

Arbor for the Summer Meeting, September 10–14. Finally Professor R. E. Langer of the University of Wisconsin paid a tribute to the life and work of Professor E. B. Skinner who had regularly attended the Chicago meetings almost from the time that they were started.

Friday morning Professor A. A. Albert of the University of Chicago presided at the Algebra Section, and Professor W. H. Bussey of the University of Minnesota at the Section on Analysis and Geometry. At the Symposium Lecture Professor D. R. Curtiss of Northwestern University presided. Professors E. P. Lane and G. A. Bliss of the University of Chicago alternated as chairmen at the General Session on Saturday morning.

The titles of papers read at the meeting follow. Those whose abstract numbers are followed by t were read by title. Papers numbered 1 to 9 were given in the Algebra Section, those numbered 10–19 in the Analysis Section, and those numbered 20 to 37 in the General Session. Dr. M. S. Robertson was introduced by Professor G. A. Bliss. Mr. L. R. Wilcox was introduced by Professor J. R. Kline.

1. *Classes of maximum numbers associated with certain symmetric equations in n reciprocals*, by Professor H. A. Simmons and Mr. William Block. (Abstract No. 41-5-228.)

2. *Metabelian groups and trilinear forms*, by Professor H. R. Brahana. (Abstract No. 41-5-229.)

3. *On ideals in generalized quaternion algebras and Hermitian forms*, by Professor C. G. Latimer. (Abstract No. 41-3-140.)

4. *On the characteristic values of the matrix* $f(A, B)$, by Professor W. E. Roth. (Abstract No. 41-3-168.)

5. *Generalized determinants of Vandermonde*, by Professor I. A. Barnett, Dr. C. W. Mendel, and Mr. Haim Reingold. (Abstract No. 41-5-230.)

6. *Maximal orders in rational cyclic division algebras of odd prime degree*, by Dr. Ralph Hull (National Research Fellow). (Abstract No. 41-3-169.)

7. *On the composition of quadratic forms*, by Dr. E. D. Jenkins. (Abstract No. 41-5-231.)

8. *A functional equation of certain Vandiver polynomials*, by Mr. H. F. S. Jonah. (Abstract No. 41-5-256.)

9. *Characteristic numbers of trilinear forms*, by Dr. Rufus Oldenburger. (Abstract No. 41-3-177.)

10. *A boundary value problem associated with the calculus of variations.* II, by Dr. W. T. Reid. (Abstract No. 41-5-232.)

11. *Analytic functions star-like in one direction*, by Dr. M. S. Robertson (National Research Fellow). (Abstract No. 41-5-233.)

12. *Local Betti numbers*, by Mr. H. E. Vaughan. (Abstract No. 41-3-179.)

13. *Concerning symmetrical cut-sets*, by Professor Wayne Dancer. (Abstract No. 41-5-234.)

14. *A Dirichlet problem for an ellipsoid lying between two infinite planes*, by Professor I. S. Sokolnikoff and Dr. E. S. Sokolnikoff. (Abstract No. 41-3-167.)

15. *Pseudominimal hypersurfaces and harmonic functions*, by Professors Otto Laporte and G. Y. Rainich. (Abstract No. 41-5-235.)

16. *Linear connections in normal space*, by Mr. C. B. Tompkins. (Abstract No. 41-5-236.)

17. *Pairs of surfaces in five-dimensional space*, by Mr. L. R. Wilcox. (Abstract No. 41-5-237.)

18. *On certain properties of Fréchet L-spaces*, by Dr. E. W. Miller. (Abstract No. 41-5-248.)

19. *Note on the factorization of Fredholm kernels*, by Professor H. P. Thielman. (Abstract No. 41-5-249.)

20. *Normal division algebras of degree p^e over F of characteristic p*, by Professor A. A. Albert. (Abstract No. 41-5-238.)

21. *Covariants of r-parameter groups*, by Professor C. C. MacDuffee. (Abstract No. 41-5-239.)

22. *Abnormality in the calculus of variations*, by Professor G. A. Bliss. (Abstract No. 41-5-240.)

23. *The isoperimetric inequality on the unit sphere*, by Professor Tibor Radó. (Abstract No. 41-3-166.)

24. *A canonical power series expansion for a surface*, by Professor E. P. Lane. (Abstract No. 41-1-100.)

25. *On locally connected spaces*, by Professor R. L. Wilder. (Abstract No. 41-3-178.)

26. *Addition theorem of Euler's* Γ*-function*, by Professor Wilhelm Maier. (Abstract No. 41-5-255.)

27. *Asymptotic solutions of certain linear differential equations in which the coefficient of the parameter may have a zero* (preliminary report), by Mr. Henry Scheffé. (Abstract No. 41-5-257.)

28. *On the Hilbert irreducibility theorem*, by Professor A. A. Albert. (Abstract No. 41-5-241-*t*.)

29. *Simple Lie algebras over a non-modular field*, by Professor A. A. Albert. (Abstract No. 41-5-242-*t*.)

30. *Amicable numbers*, by Mr. E. B. Escott. (Abstract No. 41-5-243-*t*.)

31. *On certain generalizations of the Cauchy-Taylor expansion theory*, by Dr. Gertrude S. Ketchum. (Abstract No. 41-5-244-*t*.)

32. *Expansions of functions holomorphic in distinct regions*, by Dr. P. W. Ketchum. (Abstract No. 41-5-245-*t*.)

33. *Plane sections through an asymptotic tangent of a surface*, by Professor E. P. Lane. (Abstract No. 41-1-101-*t*.)

34. *On the bilateral equation in matrices*, by Professor W. E. Roth. (Abstract No. 41-1-102-*t*.)

35. *Linear difference equations containing a parameter*, by Professor W. J. Trjitzinsky. (Abstract No. 41-5-246-*t*.)

36. *Concerning polar singularities of an analytic function*, by Dr. E. W. Miller. (Abstract No. 41-5-247-*t*.)

37. *Cyclotomy when e is composite*, by Professor L. E. Dickson. (Abstract No. 41-5-258-*t*.)

M. H. INGRAHAM,
Associate Secretary

BESICOVITCH ON ALMOST PERIODIC FUNCTIONS

Almost Periodic Functions. By A. S. Besicovitch. Cambridge, University Press, 1932. xiii+180 pp.

The seven years which elapsed between the time of creation of the theory of almost periodic functions by H. Bohr in 1925 and publication of Besicovitch's book, were years of remarkable development. The notion and properties of almost periodic functions, either in their initial, or in generalized form turned out to be of great importance in various fields of analysis, function theory, topology, and applied mathematics. The necessity of a monograph giving a concise and systematic exposition of the fundamentals of the theory of almost periodic functions was becoming more and more obvious. The task of writing such a monograph was an arduous one, but it is not astonishing that the author has completely succeeded in solving this difficult problem. The present monograph is but one of the numerous important contributions of the author to the theory of almost periodic functions for which he was awarded an Adams prize of the University of Cambridge in 1931. According to his own statement the author did not intend to give an encyclopaedic account of the theory; on the other hand, he was extremely painstaking in giving a clear and complete picture of the logical structure of the theory.

The book contains three chapters and an appendix between Chapters 2 and 3. Chapter 1, *Uniformly almost periodic functions* (66 pp.), is devoted to a discussion of continuous almost periodic functions as introduced initially by H. Bohr. The central point of the chapter is of course the proof of H. Bohr's "Fundamental Theorem," $\sum |A_n|^2 = M\{|f(x)|^2\}$. The initial proof of this theorem as given by H. Bohr was very complicated. Several other proofs were given later (Weyl, Wiener, de la Vallée-Poussin). The author gives the simplest of these proofs, which is due to de la Vallée-Poussin, and is based on a combination of ideas of Weyl and Bohr. Considerable attention is given also to the problem of summation of the Fourier series of an almost periodic function by means of sequences of partial sums, and also by means of the associated Fejér-Bochner trigonometric polynomials. A special paragraph is devoted to a discussion of arithmetical nature of translation numbers of an almost periodic function. The author considers the set $\overline{E}_\epsilon = \overline{E}(\epsilon; f(x))$ of all integers which are ϵ-translation numbers of $f(x)$, introduces the notion of almost periodicity of such sets, and proves that $\overline{E}_\epsilon$ is almost periodic for almost all values of ϵ. This theorem, due to the author jointly with H. Bohr, is interesting in itself and also plays an important role in the subsequent development of the theory of generalized almost periodic functions. The chapter closes with a rapid sketch of uniformly almost periodic functions of two independent variables.

Chapter 2, *Generalization of almost periodic functions* (56 pp.), treats of various generalizations of the notion of an almost periodic function. These generalizations, in the increasing order of their generality, are as follows: (S^p=Stepanoff) a.p. $\subset$ (W^p=Weyl) a.p. $\subset$ (B^p=Besicovitch-Bohr) a.p., $p \geqq 1$. They all include as the most special case the (U=uniformly) a.p. functions of Bohr. While (S^p) and (W^p) a.p. functions represent a "natural" generaliza-

tion of the (U) a.p. functions, which are obtained by merely modifying the metric of the underlying function space, the notion of the (B^p) a.p. involves a more radical and farther distant modification. It is significant that the recent generalizations of the theory of almost periodic functions of v. Neumann and Bochner, while being fully capable of including the theory of the (S^p) and (W^p) a.p. functions, appear to be less effective for the theory of the (B^p) a.p. functions. Let $\mathfrak{A}$ be the set of all finite trigonometric polynomials and let C_G denote the operation of taking the closure in a space $G=U$, S^p, W^p, or B^p. The central point of the theory consists in showing that for each choice of G, $(G)=C_G(\mathfrak{A})$. The relation $C_G(\mathfrak{A})=C_G\{C_U(\mathfrak{A})\}$ turns out to be of fundamental importance. Fejér-Bochner trigonometric polynomials are shown to be as efficient for summation of the Fourier series of a function of any (G) as they were in the special case of $G=U$. The climax of the whole theory is reached at the end of the chapter when the author states and proves the Riesz-Fischer Theorem: *To each formal series* $\sum A_n e^{\Omega_n x}$ *with* $\sum |A_n|^2<\infty$ *there corresponds a* (B^2) *a.p. function which has this series as its Fourier series.* This theorem does not hold true for the (U), (S^2) or (W^2) a.p. functions, although the converse theorem (Parseval's Theorem) holds for each admissible G. In the following appendix (17 pp.) the author discusses a modified notion of the $(\overline{B})$ a.p. functions which is not so general as that of (B) but possesses some advantages of its own.

The last chapter, Chapter 3, *Analytic almost periodic functions* (43 pp.), is devoted to an exposition of the theory of analytic functions almost periodic in a strip, due largely to H. Bohr. The author does not give much space to the discussion of the problem of distribution of values of such functions. On the other hand he adds several important contributions of his own, among which we shall mention only two. (i) An extension of Picard's Theorem to a general class of functions uniformly a.p. in a half-plane. (ii) A proof of the remarkable theorem to the effect that if a function is uniformly a.p. in an open strip (α, β) and can be continued analytically into a larger open strip (α_1, β_1), then, under certain restrictions of a very general form, the extended function will still be (B^2) a.p. in (α_1, β_1).

The exposition, although condensed, is extremely clear and suggestive throughout the whole book. The author has succeeded in his endeavor to make his book self contained by eliminating as many references to other sources as possible. Thus he inserted proofs of various auxiliary propositions necessary for the general discussion, which will be of greatest help to the reader. Misprints are not entirely non-existent in the book, but their number is very small. It is curious that the name of Bohl, who introduced the notion of quasi-periodic functions is confused with that of Bohr, and that the name of F. Riesz is consistently spelled as F. Riesc.

Taken as a whole, the *Almost periodic functions* of Besicovitch represents a notable contribution whose study is indispensable to every one working in the field of almost periodic functions, and which is destined to become a starting point of many an important investigation in this fascinating theory.

J. D. TAMARKIN

BROWN AND SHOOK—PLANETARY THEORY

Planetary Theory. By E. W. Brown and C. A. Shook. Cambridge, University Press, 1933. xii+229 pp.

The reviewer's lot is not "a 'appy one". He is expected to give in a brief measure of time and space a critical examination of a treatise covering several hundred pages and representing many years of investigation by renowned authors. And above all he is expected to make some contribution to the errata of the text! Realizing the un'appiness of the lot before him, the reviewer proceeds with much appreciation and with no little trepidation in an effort to summarize rather than to examine critically. And he has no contribution to make to the errata! The text already contains a page of such, contributed by experts, and that is sufficient for any volume.

When William Thomson (afterwards Lord Kelvin) and his older brother James accompanied their father on a summer tour through Germany, it is recorded that William, then 16 years of age, took Fourier's famous *Analytical Theory of Heat* for light reading. The reviewer does not recommend *Planetary Theory* for any such journey except possibly to those with Kelvin's I.Q.

"The purpose of the volume", as stated by the authors, "is the development of methods for the calculation of the general orbit of a planet". It does not aim to be a substitute for a treatise like Tisserand's or Laplace's nor does it contain detailed accounts of such classical theories as are to be found in Newcomb or elsewhere.

The mathematical processes are largely formal. Rigor is desirable when attainable, but results are much more to be desired (compare *Proverbs* on a *good name* and *riches*), particularly when the results appear to be reasonable and "useful for the prediction of physical phenomena".

Considerable portions of the volume are new, particularly the work on resonance and the Trojan asteroids.

Various forms of the equations of motion are derived in Chapter 1. For the development of planetary theory the *osculating plane* possesses certain advantages as a principal plane of reference. It is the plane passing through the sun and tangent to the orbit of the planet. Its motion, being either slow or small, affects the motion of the planet in a way which can be quite simply accounted for or neglected entirely. Two systems of coordinates may be used with the osculating plane as the principal plane of reference. Three variables are used in one system and six in the other. In the latter case the variables are the elements of the osculating ellipse. Polar coordinates are next used. The equations are then put into canonical form and also derived first with the true orbital longitude and later with the disturbed eccentric anomaly as the independent variable. The chapter concludes with the derivation of the equations of motion referred to the coordinates of the disturbing planet and also to any arbitrary plane of reference.

Devices are treated in Chapter 2 to simplify the expansions of various functions into sums of periodic terms. Lagrange's well known theorem for the

expansion of a function defined by an implicit equation is considered and extended to three variables. Fourier series, functions of Fourier series, Bessel functions of the first kind, and hypergeometric series are next treated. The chapter concludes in an encouraging way with devices to relieve fatigue in calculation with slowly converging series.

The third chapter deals with elliptic motion in which the eccentricities are sufficiently small for numerical calculation without too much labor. Instead of using the three anomalies directly, namely, the true, the eccentric, and the mean, which are denoted by f, X, and g, respectively, use is made of

$$\phi = e^{if}, \ \chi = e^{iX}, \ \psi = e^{i\overline{X}}, \quad i = \sqrt{-1}.$$

Fourier developments are obtained for the radius or powers of the radius in terms of ϕ, χ, ψ, and also f and g. In certain of these expansions Bessel functions or hypergeometric series occur. The chapter concludes with certain detailed developments for g, f, r to be used for reference, and with numerical developments by harmonic analysis.

Chapter 4 deals with the development of the disturbing function, R. When the angle, I, between the orbital planes of the two planets is zero and the eccentricities e, e' are so small that they can be neglected, R can readily be expanded as a Fourier series in the coefficients of which occur powers of $\alpha = a/a'$, a, a' being the mean distances of the planets from the sun. When e, e', I are not neglected, R can be expanded in powers of these parameters, and these expansions converge rapidly as e, e', I are usually small. Difficulty occurs, however, when through integrations small denominators arise sometimes involving discontinuities.

In the older methods the time is used as the independent variable, which requires the disturbing function to be expressed in terms of the three anomalies. The methods developed in this chapter involve the theorem

$$F(p^D, x) = p^D F(x), \ D = x\frac{d}{dx},$$

where F is expansible in integral powers of its argument, and p^D is expansible in positive integral powers of D. By the use of this theorem, expansions for the distances between the planets are obtained in powers of the eccentricities and multiples of the true anomalies; and also in powers of the inclination and multiples of the true anomalies. Transformations are then made to obtain similar developments in multiples of the mean anomalies. To obtain the development in terms of the eccentric anomalies use is made of a hypergeometric series in which certain of the elements are linear functions of the symbolic operators D, B, B', where

$$D = \alpha\frac{\partial}{\partial\alpha}, \quad B = \chi\frac{\partial}{\partial\chi}, \quad B' = \chi'\frac{\partial}{\partial\chi'}.$$

Newcomb gave a somewhat similar expansion in terms of the eccentric anomalies of which certain portions were carried out to the seventh order with respect to the eccentricities, but he did not obtain a general formula like the one developed here which permits any coefficient to be written down at once.

The chapter ends with certain devices to abbreviate and to check the computation. For example, it is shown that after transformations have been made so as to calculate easily and readily two coefficients in a series, all the remaining coefficients and their derivatives can be deduced from these two by the use of finite formulas.

The fifth chapter contains the elements of the theory of canonical variables so far as it is needed in the later work. New symbols d, δ are suitably defined, and on making use of them the equations

$$\frac{dx_i}{dt} = \frac{\partial H_i}{\partial y_i}, \quad \frac{dy_i}{dt} = -\frac{\partial H_i}{\partial x_i}, \qquad i = 1, 2, \cdots n)$$

can be expressed in the form

$$\sum_i (dx_i\,\delta y_i - dy_i\,\delta x_i) = dt\delta H.$$

The Jacobian transformation can then readily be made from one set of variables to another set. It is shown that a general solution of the canonical equations written above is provided by the equations

$$y_i = \frac{\partial S}{\partial x_i}, \quad p_i = \frac{\partial S}{\partial q_i},$$

where p_i and q_i are arbitrary constants and S is an integral of the partial differential equation

$$H\left(x_i, \frac{\partial S}{\partial x_i}, t\right) + \frac{\partial S}{\partial t} = 0.$$

Applications are made showing various sets of canonical elements of elliptic motion.

In Chapter 6 it is shown how the theory of canonical equations can be applied to the calculation of the orbit of a planet. "All the methods depend fundamentally on the assumption that the variables differ from constants by amounts which have as factor the ratio of the disturbing mass to that of the primary, and therefore that the variables may be developed in powers of this ratio." Delaunay's method is adopted, namely, to obtain a change of variables such that the new variables are more nearly constant than the old ones. The main difference is that while Delaunay made numerous changes, the present authors show that, in general, one change is sufficient for the solution of the majority of planetary problems. "Much of the discussion in the chapter hinges on the amount of labour which the development and solution of the equations for the new variables require."

It is first shown how the work may be so adapted that use can be made of the developments for R in Chapter 4 which are not in terms of canonical elements. The disturbing function is then split up into two parts $R = R_s + R_c$, and when these are suitably defined, differential equations are obtained which have the same form as the original equations except that the terms in R_s have disappeared. Two choices for the division of R are made. In one R_c contains only those terms which produce the so-called secular motion of the elements. In the other, R_s contains the short period terms only while R_c contains the long period and secular terms. It is assumed that R_c is expansible in powers of

m', the mass of the disturbing planet, and the solutions of the above-mentioned differential equations are readily obtained to the first degree of approximation. These solutions are called the *perturbations of the elements*. The secular, periodic, and long-period terms in the first approximation are next considered.

In the calculation of the values of the variables to the second degree of approximation, special attention is given to discover what classes of terms will give sensible changes to the expressions found in the first approximations.

At the conclusion of the chapter the advantages and disadvantages of the theory which has been developed are pointed out. It has a "simplicity of analytical form" which makes it attractive for many theoretical investigations, particularly those concerned with resonance, but because of the labor involved and the slow convergence of certain series, it is doubtful if it is the most convenient for the calculation of ordinary planetary perturbations.

The direct calculation of the coordinates is developed in Chapter 7 with the true orbital longitude as the independent variable. The solutions of the equations of motion are obtained by continued approximation. The first approximation with $m'=0$ gives elliptic motion and the solution thus found is called the *elliptic approximation*. To obtain the next approximation, called the *first approximation*, the expressions found for the elliptic approximation are substituted in the terms having m' as a factor and the resulting equations are solved, giving the *second approximation*. Higher approximations can be obtained by continuing the process, but it is rarely necessary in planetary problems to proceed beyond the first approximation except for those terms which on account of their long periods have received small denominators through integration.

After the first approximations are obtained the "equations for the variation of the elements" are considered before the second approximations are treated. In the variation of the elements three new variables are introduced for two of the old variables. This procedure gives equations more convenient for calculation and furnishes an important theorem concerning the secular terms. In this way separate calculations can be made of the effects due to the secular terms and to the periodic terms in the first approximations. So as to compare the theory developed in this chapter with other theories, particularly the theory developed by Hansen, a transformation to the time as the independent variable is made. Approximate formulas for the perturbations, final definitions, and the determination of the constants conclude the chapter.

"Chapter 8 contains an attempt to place the theory of resonance on a general basis in a form which permits of application to specific problems. It appears to give a method of approach to the consideration of the question of general stability of the orbits of the planets."

Resonance is defined as a case of motion in which a particle or body, moving or capable of moving with periodic motion, is acted on by an external force whose period is the same as that of the motion of the body. The usual illustration is the equation

$$\frac{d^2x}{dt^2} + n^2x = m \sin n't,$$

which yields a Poisson term in t explicitly when $n'=n$. It is pointed out, how-

ever, that the illustration is defective inasmuch as the particular integral has a very large coefficient when n' is near n. Further, in actual mechanical problems, x does not always occur linearly and, in addition, it is sometimes present in the expression for the disturbing forces.

In order to illustrate the principal features of certain of the resonance problems, use is made of the motion of a pendulum which can make complete revolutions about a horizontal axis as well as oscillate about the vertical and which is disturbed by a periodic force. In certain cases, however, the analogy breaks down and special devices have to be employed to find out whether resonance is possible. The general case of resonance in the perturbation problem is next considered, and while the investigation given does not prove the existence of resonance it at least shows that so far no condition preventing it has appeared. The chapter concludes with a method of procedure applicable to certain of the actual cases of resonance in the solar system, particularly in the case of the asteroids disturbed by Jupiter and Saturn.

Chapter 9 is devoted to a consideration of the motion of the Trojan group of asteroids. These are two groups which are to be found at the vertices of the equilateral triangles described on the line joining the sun and Jupiter as base and in the plane of Jupiter's orbit. They oscillate about these triangular points of libration in much the same way as their mythological namesakes circulated about the walls of ancient Troy. The libration points themselves were first discovered by Lagrange in 1772 and were considered by him as "pure curiosities". The first asteroid in the group was discovered in 1901 and the last one, the tenth, in 1932.

The development in this chapter is much the same as that used in Chapter 6. The disturbing function R is expressed in powers of the eccentricities and mutual inclination. New variables are then introduced which leave the equations of motion canonical and at the same time eliminate the short period terms from R. Solutions for these variables are then obtained in terms of the time.

Finally, the perturbations due to Saturn are considered. In the ordinary planetary theory the procedure would consist in finding the perturbations due to each planet separately, then those due to their combined action, and adding the results. This method is not applicable in this case, however, inasmuch as the action of Jupiter cannot be neglected even in finding a first approximation of the direct action of Saturn. It is assumed that the mutual perturbations of Jupiter and Saturn on each other are completely known. Hence, when considering the direct effect of Jupiter upon the asteroid, the indirect effect of Saturn in causing Jupiter to deviate from elliptic motion must also be taken into account, and likewise the indirect effect of Jupiter upon Saturn. It is shown that Jupiter so alters the direct effect of Saturn upon the asteroid that by far the largest part of the action of Saturn is indirectly through its perturbation on Jupiter.

The appendix on *Harmonic Analysis* contains formulas for application to the development of a given function in a form ready for actual use.

DANIEL BUCHANAN

TWO BOOKS ON TOPOLOGY

Vorlesungen über die Theorie der Polyeder unter Einschluss der Elemente der Topologie. By Ernst Steinitz; edited and completed by Hans Rademacher. Berlin, Springer, 1934. viii+351 pp.

Lehrbuch der Topologie. By H. Seifert and W. Threlfall. Leipzig and Berlin, Teubner, 1934. vii+353 pp.

These two books present an interesting contrast; both have to do with topology in a pure, or combinatorial form, but otherwise they have little in common. The one is a memoir; the other is a textbook. The one is an isolated chapter of geometry which bears little relation to the main stream of contemporary topological research, but which stands by itself, firm in its own intrinsic worth. It deals with a question almost as old as analysis situs itself—the combinatorial classification of ordinary polyhedra. The other is an exposition of the rudiments of modern topology—the homology theory of n-dimensional complexes and manifolds. It aims to give the student a glimpse of the far-reaching developments of present-day topology, and to acquaint him with the vital ideas at their root.

The book by Steinitz is a posthumous memoir, appearing as the forty-first volume of the familiar yellow-covered series put out by Springer. It carries on the study of ordinary (two-dimensional) polyhedra already begun by Steinitz in his *Enzyklopädie* articles on *Polyeder und Raumeinteilungen*. The theory develops about a single central problem: Are there necessary and sufficient combinatorial conditions in order that it be possible to realize geometrically a two-dimensional complex by a convex polyhedron? In other words, can convex polyhedra be characterized in a purely combinatorial fashion? This problem has a classical ring; it is a natural outgrowth of the pioneer work of Euler on convex polyhedra; yet here it receives for the first time a complete and definite affirmative answer. The book is self-contained, well-illustrated, and easy to read. One cannot go far in it without sensing the enthusiasm of an author who has made his subject a hobby as well as a serious piece of mathematics. Steinitz' fame may rest on his contributions to algebraic field-theory, but he evidently took just as much delight, if not more, in working with polyhedra.

The book is divided into three parts. The first part makes an exploration, partly historical and partly intuitive, of possible forms and types of polyhedra. It discusses the following topics: the Euler formula $(v-e+f=2)$ and its extensions, polyhedral volume, the topological forms of surfaces (including the non-orientable), the Cauchy theorem on the rigidity of a convex polyhedron, and the Legendre determination of the number of degrees of freedom involved in the construction of a polyhedron of given type. At the end of this preliminary survey the problem of the combinatorial characterization of convex polyhedra is broached through polyhedra with triangular faces and the dual polyhedra with three-edged vertices.

The second part of the book presents a purely combinatorial theory of polyhedra, guided by the preceding exploration, but founded on strict abstract

principles. A (two-dimensional) *complex* is defined as an abstract collection of "vertices," "edges," and "faces," bound together by specified "incidences." Selective restrictions are gradually imposed until complexes evolve which are regular enough to represent surfaces, then polyhedra, and finally convex polyhedra. The combinatorial counterpart of a closed surface, or a surface bounded by a number of non-intersecting curves, is a *normal complex* (known to topologists as a manifold). If two normal complexes have the same Euler characteristic and the same number of boundaries, and if both are orientable or non-orientable, it is possible to pass from the one to the other by a sequence of moves which split a face or edge in two or which perform an inverse operation, thus adding or subtracting an edge and either a face or a vertex. This is a combinatorial form of the well known classification of (two-dimensional) manifolds. A complex is called a *polyhedron* if it is normal except for singular vertices, and if it does not have more than one edge jointly incident with two faces or with two vertices. Such a polyhedron is said to be without *encroaching elements* if it possesses a pair of faces with two vertices in common only if the vertices are the end-points of a common edge. By appropriate composition of the splitting moves mentioned above it is possible to get "regular" moves which leave invariant the notions of *polyhedron* and *polyhedron without encroaching elements*. The combinatorial theory culminates in the *K-polyhedron*, which is both an Euler complex (that is, a normal complex with characteristic two) and a polyhedron without encroaching elements. A *K-polyhedron* possesses the important characteristic that it can be derived from a tetrahedron by regular splitting.

The third part of the book gives proofs, which are rigorously based on the foundations of geometry, that each K-polyhedron can be realized by a concrete convex polyhedron. Two of the proofs depend on the fact that a K-polyhedron can be derived from a tetrahedron by regular splitting. The first proof uses analytical considerations of linear independence to establish this, while the second uses purely geometrical considerations of incidence and order (without continuity). The third proof is projective; it also uses considerations of incidence and order (without continuity), but it depends on certain interesting new types of moves.

We are indebted to Rademacher for the complete and polished form in which this book comes to us. Steinitz had not finished the manuscript when he died in 1928. Rademacher had to fill a gap in the second part of the memoir and to write the last half of the third part.

The textbook of Seifert and Threlfall should do much to smooth the path of the student who wants to learn the fundamentals of (combinatorial) topology. The authors have concentrated on basic concepts and methods, avoiding generalizations; they have explained these concepts and methods in as simple and concrete a fashion as possible, yet one which is thorough and rigorous. The exposition proceeds by easy stages, with examples and illustrations at every turn. It presupposes nothing; even the necessary group theory is developed in a special supplementary chapter.

The book follows the lines laid down in Veblen's *Analysis Situs* (1922, 2d ed. 1931), van Kampen's dissertation (1929), and the early chapters of Lef-

schetz' *Topology* (1930). The influence of American topologists has been so strong that terminology hitherto confined to English has been translated; for example, "chains" finally appear as "Ketten." The influence of the German algebraic school is reflected in a frank use of group theory. A particularly valuable feature of the book is the attention paid to the fundamental (Poincaré) group, covering spaces, and three-dimensional manifolds; in no other single place in the literature has so much interesting information been gathered together on these topics. The investigations of the authors themselves find place in the chapter on three-dimensional manifolds.

The first two chapters are largely preparatory; the first draws on intuition to create a suitable topological atmosphere, the second defines the geometrical foundations on which the subsequent work is built. The idea of neighborhood is introduced to permit the definition of a continuous mapping of one figure on another (any neighborhood on the second figure contains the map of a neighborhood on the first). Then simplexes appear, and complexes are defined as "neighborhood-spaces" which can be cut into simplexes.

Chapters 3–5 deal with the fundamental topological invariants of an n-complex: Betti numbers, torsion coefficients, dimension. In developing the homology theory the authors give simplicial chains precedence over singular ones; they turn to the latter only when it is necessary to establish the topological invariance of the homology groups of a complex. This is the customary procedure, but the reviewer feels that if it were reversed the theory would gain a better balance and readier comprehension. The notion of singular chain, if taken at the beginning, would no longer seem at all artificial, and simplicial chains would follow naturally as a special case. Instead of the usual invariance theorem one would have the important fact that the homology groups of a complex can be reduced to groups with finite simplicial bases.

The authors confine their attention to chains and cycles with integral coefficients or coefficients mod 2, but they give plenty of illustrative material; homology groups are worked out in full for several sample complexes. Despite the criticism above it must be admitted that the proof of the topological invariance of the homology groups of a complex is well done; appropriate emphasis is placed on the part played by deformations and simplicial approximation of continuous mappings. Further invariance proofs use local homology groups to establish the permanence of dimension, and other properties of a complex. The local groups appear again in an essential fashion when manifolds are defined.

The last six chapters of the book (excluding the supplementary chapter on group theory) treat topics connected with manifolds—first two-dimensional, then three-dimensional, and finally n-dimensional. The two-dimensional manifolds are classified by the methods of Brahana (although the text does not attribute them to him). Then, after a digression on the fundamental (Poincaré) group of a complex and a further digression on covering complexes, there is a chapter on methods of constructing and analyzing three-dimensional manifolds.

A (closed) n-dimensional *manifold* is defined to be a connected n-complex at each point of which the local homology groups are those of an $(n-1)$-sphere; this definition expresses the essential combinatorial character of a manifold in a topologically invariant manner. The Poincaré duality theorem is estab-

lished, as well as the standard theorems on intersection and linking numbers. A final chapter deals with the Brouwer degree of a mapping, and the Lefschetz fixed point formula treated by the methods of Hopf.

The book contains a very complete bibliography, it is well indexed, and as a further help to the reader most chapters are prefaced by a summary of their contents. At many points in the text the reader is referred to supplementary notes collected at the end of the book, which indicate extensions of the theory and links with other work. Little attempt is made, outside these notes, to attribute ideas to their originators; this is particularly glaring when the ideas are heavily exploited. The authors apologize for their failure to treat the Alexander duality theorem and the theory of compact metric spaces (so beautifully rounded out by the recent work of Pontrjagin), but they promise to write a second volume on these matters if someone else does not do so first.

A. W. TUCKER

WEDDERBURN ON MATRICES

Lectures on Matrices. By J. H. M. Wedderburn. American Mathematical Society Colloquium Publications, volume 17. New York, American Mathematical Society, 1934. vii+200 pp.

For the past seventy-five years matric theory has been growing in stature and in favor among men. Many branches of mathematics have been more promising infants, but few have shown such sustained growth and ever widening field of application. The concept of matrix, like that of group, extends its roots under algebra, number theory, geometry, differential and integral equations, wave mechanics—a fairly representative cross-section of modern mathematics. This fundamental nature of matric theory has never been so generally appreciated by mathematicians as at present. Thus Wedderburn's book is timely.

An evident fact in the history of matric theory is that the important theorems are not due to any small group of men. A few names stand out prominently, of course, but it has taken close to a thousand distinct contributions to bring the theory to its present state. Why this was so is not evident, but it must be true that the theorems which now seem so clear to us were not intuitive to mathematicians at the time of their discovery. It has been the common history of the important theorems that they were discovered first in special cases, then generalized and laboriously proved, and finally furnished with neat direct proofs. In the book under review most of the theorems have reached the last stage of development, and the reader is apt to be unaware of the amount of publication which it renders obsolete.

In recent years Wedderburn has been one of the most important contributors to matric theory. His discoveries have been published as they came, and have taken their place in matric lore. It would be absurd, therefore, to expect a large proportion of the results in this book to be new. But theorems have been extended, sharpened, and clarified to a remarkable degree.

It is the organization and presentation of the material, however, which

make the peculiar appeal of the book. This is no mere compendium of results—the subject has been completely reworked and the proofs recast with the skill and elegance which come only from years of devotion.

The treatment is primarily that of the British school. The matrix is introduced through its representation as a linear vector function, and treated alternatively from this point of view and abstractly as an element of a matric algebra, according to convenience. Vector analysis, matric theory, and linear associative algebra are all integrated in this treatment, and each is used to advantage in proving results in the others. The reviewer's chief regret is that the theory was not brought into closer harmony with the modern German school of algebra; in many cases only a rewording is necessary to bring this about.

Chapter 1 is concerned with vectors, linear vector functions, basis, adjoint, and rank. Chapter 2 treats matric polynomials, the characteristic equation, and the square root of a matrix. In Chapter 3 we find the all-important theory of invariant factors and elementary divisors. Chapter 4 is devoted to the theory of vector polynomials and the equivalence of pencils of the type $A\lambda+B\mu$. The author gives his own presentation of this rather difficult topic, and while there is no great gain in brevity, the treatment seems to be rigorous and complete.

In Chapter 5 we have a treatment of compound matrices, direct products, power matrices, and a neat treatment of Schur's associated matrices. Chapter 6 is on hermitian, unitary, and orthogonal matrices with several nice proofs by the author. Chapter 7 is on commutative matrices, and includes a sharpening of the famous theorem of Frobenius concerning the characteristic roots of $\psi(x, y)$, where x and y are commutative.

Chapter 8, under the title of functions of matrices, contains many interesting morsels. The determinations of a multiform function of a matrix are well treated, with examples. We also find an amplification of the author's theory of the absolute value of a matrix, generalizing Hadamard's theorem on the maximum value of a determinant. Then come the derivative and integral of a matrix, matric functions of a scalar variable, then functions of a variable vector, and functions of a variable matrix. These lead to the covariant derivative of a tensor and to quantum matrices of finite order.

Chapter 9, on the automorphic transformation of a bilinear form, was first published by the author in 1922 in but slightly different form.

The last chapter is an elegant treatment of linear associative algebras, the subject which was started on its modern development by the publication of the author's dissertation in 1907. The linear algebra is a natural generalization of the matric algebra as the author has presented it, so the inclusion is a natural one. The principal decomposition theorems have here been materially simplified.

In brief, this book consolidates the gains of the last seventy-five years, filling in gaps and simplifying proofs, and laying a firm foundation on which the matric theory of the next century will arise.

C. C. MacDUFFEE

SHORTER NOTICES

Einfuehrung in die Hoehere Analysis. By Ernst Lindelöf. German edition by Egon Ullrich. Leipzig, Teubner, 1934. ix+526 pp.

This volume fulfills in an able manner the promise of its title, that of giving an introduction to higher analysis, which may serve either as a text in a class, or be used for self-instruction. It reads like a series of lectures given by an able teacher with a wide acquaintance with the field presented and a long experience in the presentation of material. It might well serve as a model of exposition even for the teacher of calculus of long standing. Among the points to be noticed are the frequent occasions when the author presents a point from two or three different angles. It is further remarkable how well the author has succeeded in stating the fundamental theorems in the calculus in such a way as not to sacrifice in accuracy, and still be intelligible. The beginning of the chapter on the definite integral, where care is taken to define the meaning of length and area, is a case in point. On the other hand, where necessary, the author is content with stating the theorem and delaying proof until greater maturity has been reached, as for instance in properties of continuous functions, which are taken up in a chapter near the end of the book.

The beginning sections of the book bring, in addition to an admirable introduction to elementary functions, and the treatment of limits, a chapter on approximate computation, with special attention to the question of significant figures, and a chapter on continued fractions. The principal section of the book is devoted to the essentials of differential and integral calculus, with many delightful sidelights. Near the close of the book, the author indulges in a chapter on real numbers, which is really a fine introduction to the theory of functions of a real variable, and a chapter on complex numbers which includes the general solution of the cubic and biquadratic equations. An appendix brings an introduction to the theory of determinants, the presentation of which suggests the possibility of defining the expansion of determinants by expansions by minors, using an induction process for proof, a notion which might well have advantages for an introduction to the subject. Another appendix lays the foundation of the treatment of areas by developing the formula for the area of a closed polygon

$$\sum_{i=1}^{n} \tfrac{1}{2}(x_i y_{i+1} - x_{i+1} y_i);\ n+1 \equiv 1,$$

seldom found in analytic geometries, and a corresponding result for volumes of polyhedra.

One might take exception to the length of the volume, but the objective, the clearness and elegance of the treatment, make this item seem insignificant. After all, as the German editor remarks, it is not the paper and printer's ink which should be minimized, but the enjoyment of the ultimate consumer which should be maximized, and this ideal has certainly been attained in this volume.

T. H. Hildebrandt

Einführung in die Höhere Mathematik. By H. von Mangoldt. Sixth edition. Completely revised and enlarged by K. Knopp. Three volumes. Leipzig, Hirzel, 1931–33.

The *Introduction to Higher Mathematics* of von Mangoldt has held a notable place in the extensive German literature since the publication of the first edition more than twenty years ago. This was distinguished by the clarity of the exposition and the rigor of the proofs. The several editions during the life of the author testified to the value of the book as a work of reference because of its accuracy and as a text for students because of its easy comprehensibility and the inclusion of well chosen exercises. The new editions prepared by von Mangoldt showed no essential changes from the first one. When, after the death of the author, it became apparent that the work had grown somewhat obsolete, the publisher invited Professor Knopp to revise the book with the result that it has been completely rewritten.

The most extensive changes occur in the first volume which is concerned with such fundamental notions as the systems of rational, real, and complex numbers, point sets, sequences and limits, functions and continuity. The second volume includes differential calculus, infinite series, elements of differerential geometry and of functions of a complex variable. The third volume is devoted to the integral calculus and its applications, including complex variables, Fourier series, and differential equations. The list of individual topics is too long to reproduce here, but the choice of material is such as to furnish a very satisfactory preparation for the study of more advanced mathematics from the modern point of view.

W. R. LONGLEY

Leçons sur les Fonctions Presque-Périodiques. By J. Favard. Paris, Gauthier-Villars, 1933. viii+181 pp.

This book is the 13th volume of the Series "Cahiers Scientifiques, publiés sous la direction de M. Gaston Julia," and is based upon a course delivered by the author at the Collège de France. It was the author's intention to give an exposition not only of the theory, but also of some of its applications. Such are, for instance, applications to the theory of linear differential equations with almost periodic coefficients, theory of perturbations, distribution of values of analytic functions almost periodic in a strip, and of harmonic almost periodic functions.

The exposition of the theory is confined primarily to the uniform almost periodic functions of H. Bohr, with a short sketch of various generalizations in the last chapter. As to applications, the author gives mainly an account of his own investigations. His results in the theory of differential equations are suggestive and interesting; however, the reviewer can not help feeling that the whole theory is still in a somewhat primitive stage, and that a considerable amount of work will have to be done in order to bring the theory of linear differential equations with almost periodic coefficients to a level comparable to that of the analogous theory of linear differential equations with periodic coefficients. The book is well and carefully written and will prove to be of value to those desiring a rapid introduction to the theory of almost periodic functions and their applications.

J. D. TAMARKIN

Vorlesungen über Geschichte der Antiken Mathematischen Wissenschaften. Band I. *Vorgriechische Mathematik* mit 61 Figuren. By O. Neugebauer. Berlin, Springer, 1934. xii+242 pp.

The most amazing discoveries in the history of science are presented in this account of pre-Grecian mathematics. Up to this time, Egyptian papyri of considerable extent as to length and contents have been completely translated and explained. The Rhind or Ahmes Papyrus has been subjected to the most careful study as a result of advances in Egyptology and two treatises on this early historical record have appeared recently. The contents of an earlier document, the Moscow Papyrus whose existence was first really made known in 1917, have been made accessible to historians through translation only within five years. The records of Babylonian mathematics had revealed no more than certain facts about notation, a geometry of mensuration, the simplest arithmetical and geometric progressions, and other notions of the most elementary kind.

And now come these investigations with the revelation beyond the shadow of a doubt of a development of algebra by the Babylonians which raises them to the rank of real mathematicians. Fortunately records of different epochs are at hand so that this algebra is seen as a step in a growth through a long period of time. The author devotes four chapters to the role of numbers in Babylonian mathematics and to the complete number system on which their entire structure rests. It is a position system with 60 as the base and the arithmetical technique set up for whole numbers and fractions is as simple as the present day decimal system. The acceptance then of such a number system led easily to developments in other directions. Irrational numbers, and new facts in geometry appear. Among the topics treated so convincingly in this work are the solution of linear equations in more than one unknown, solution of quadratic equations, cubic equations, not yet completely understood, biquadratic equations. A single symbol is used for a concept or idea in general writing, and so it is a natural step to make use of such a symbol in algebraic problems.

Most impressive is the claim of the author with respect to the appearance of the "proof" in mathematics. To the Greeks has hitherto been given the great distinction of providing the first deductive proof in any science. This has, indeed, differentiated Grecian from pre-Grecian mathematics. This distinction must now be set aside. If a "proof" consists of a series of logical steps leading from one statement to another, then the Babylonians had a "proof." It is impossible to believe that such a complicated system of formulas as are used in the solution of certain problems (given in this work) could have been arrived at empirically.

Only passing reference can be made to the evidence that this mathematics is entirely distinct from that needed for temple and state and must have been taught in special schools; and to the growing conviction that the Arabian algebra was the outgrowth of a Babylonian algebra rather than Greek or Hindu.

This is the first time that an attempt has been made to give a complete presentation of the history of mathematics before the Greeks. Most of the chapters have already appeared in whole or in part in *Quellen und Studien zur Geschichte der Mathematik* published at intervals since 1928, and some of the

facts presented are already well known through articles and books in English. These studies, however, have been woven together into a whole at the same time that the results of more recent investigations have increased their value. A study of language and script has been added. The chapter headings cover: *Babylonische Rechentechnik; Allgemeine Geschichte Sprache und Schrift; Zahlensysteme; Ägyptische Mathematik; Babylonische Mathematik.*

This work has broadened the world for the historian of mathematics. For this, as for much more, the historian is greatly indebted to the author.

LAO G. SIMONS

Aufgabensammlung zur höheren Algebra. By Helmut Hasse. Sammlung Göschen, vol. 1082. Berlin, de Gruyter, 1934. 175 pp.

The present collection of problems is a sequel to the author's two-volume treatise on algebra in the Sammlung Göschen. The main headings are: *Theory of fields and rings, Groups, Linear algebra and determinants, Roots of algebraic equations including the solution of equations by radicals.* Throughout the author takes the point of view of abstract algebra, trying to make the problems supplement and extend the theories of his textbook. Some problems are strikingly new. It is for instance rather surprising to find given as problems Zassenhaus' proof for the generalized Jordan-Hölder theorem and Witt's proof for Wedderburn's theorem that any field with a finite number of elements must be commutative.

Some problems are quite simple, but on the whole they become increasingly difficult until one reaches the last stage at the final problem where the author confesses that he cannot solve it himself. The book may be recommended to anyone interested in abstract algebra and it should form a suitable supplement to a more advanced course in higher algebra.

OYSTEIN ORE

Mathematische Grundlagenforschung. Intuitionismus. Beweistheorie. By A. Heyting. Berlin, Springer, 1934. iv+73 pp.

This pamphlet is one of the series *Ergebnisse der Mathematik* published by the editors of the Zentralblatt. It deals chiefly with the foundations of mathematics, and mathematical logic, from two points of view, the intuitionism of Brouwer and the formalism of Hilbert, and gives an able, clear, and concise account of the essentials of these two points of view and of the important results which have been obtained in connection with them. As explained in the introduction, no attempt is made to give an account of the logistic formulation of the foundations of mathematics, a subject which is to be treated in a later number of the series.

This work is recommended, not only to mathematical logicians, but also to mathematicians in general who desire an understandable survey of its field. The reviewer knows of no better such survey, indeed of none nearly so good.

The first section begins with a notice of Poincaré as historical forerunner of intuitionism, describes the point of view of the French semi-intuitionists as they are here called (Borel, Lebesgue, Baire), the first theory of Weyl, and

the point of view of F. Kaufmann, and then gives an account of the intuitionism of Brouwer and of the results in connection with it of Brouwer, Heyting (the author of the present work), Kolmogoroff, Glivenko, Gödel, Gentzen, de Loor, Belinfante. The second section discusses the classical axiomatic method and the concepts of consistency and categoricity, then proceeds to an account of Hilbert's formal system and the Hilbert concept of a metamathematical proof of consistency. The consistency proofs of Ackermann and von Neumann are outlined; and brief mention is made of the consistency proof of Herbrand; also of the famous theorem of Gödel and its significance in this connection. And the section ends with a discussion of the relationship between formalism and intuitionism. The third section gives a description of several other points of view on the foundations of mathematics, notably those of Mannoury and Pasch. The fourth section discusses the relation of mathematics to the natural sciences, comparing the formalistic and the intuitionistic accounts of this relation. At the end of the pamphlet is a five-page bibliography of publications in this field.

ALONZO CHURCH

The Differential Invariants of Generalized Spaces. By T. Y. Thomas. Cambridge, University Press, 1934. 241 pp.

This book has a special place in the growing literature on linear displacements in a general manifold of an arbitrary number of dimensions. It deals, as the title indicates, with the differential invariants of such manifolds, thus excluding to a considerable extent special geometrical points of view, and concentrating on the analytical side of the theory. There it throws full light on a field in which the author has distinguished himself for many years through fundamental contributions. This comprehensive account of the present state of things is the more welcome as it is the first of its kind to be written.

The book begins with a general discussion of n-dimensional spaces as a basis for a theory of differential invariants, starting from a selected set of fundamental postulates. An affine connection is introduced leading immediately into the midst of things: the affine and the projective geometry of paths, Riemannian spaces, spaces with distant parallelism, conformal and Weyl spaces. The immediately following chapters give the foundations of the invariant theories of these spaces, built upon the motion of affine, projective, and conformal relative tensors. Then follow normal coordinates on which the general theory of extension of tensors is based, a theory which leads us from one relative tensor to other tensors by means of differentiation. A next chapter is devoted to an exposition of spatial identities based on the concept of the complete set of identities of the components of an invariant, that is a set of identities furnishing all the algebraic conditions these components satisfy. The simplest example is the complete set of identities of the components $g_{\alpha\beta}$ of the fundamental tensor of a metric space, which consists of the identity $g_{\alpha\beta}=g_{\beta\alpha}$. This leads to complete sets of identities for the curvature tensor of metric space, the projective curvature tensor, and to so-called divergence identities. Then follows a chapter on absolute scalar differential invariants and parameters, which can be considered as defined by means of complete systems of

linear partial differential equations. The theory is given for the case of metric space and space of symmetric affine connection, and requires an introduction into the theory of continuous groups.

This leads up to the two chapters dealing with the equivalence of generalized spaces and reducibility of one type of space to another type. Finite equivalence theorems are given for two affinely connected spaces, for spaces of distant parallelism, and for conformal spaces. Among the theorems on reducibility we mention those on the reduction of an affine space of paths to a metric space and to a Weyl space. The final chapter gives the determination in finite form of the functional arbitrariness of certain sets of quantities consistent with the existence of an affine and a metric space for which they are tensors of a special type. There is also information on the important subject of arithmetic invariants of spaces.

This short account of the contents of the book may perhaps give an impression of the rich variety of subjects, some old, many new, which are discussed by the author. It will, probably for a considerable time, be the standard work of reference on problems relating to the differential invariants of manifolds with a linear connection. But not only those interested in the special research of the author will find in this book material of importance. There is also much worth reading on the theory of partial differential equations, on group theory, and on differential geometry. The explanation of the subjects is quite clear, and the use of small print for those passages which only supplement the main ideas of the text emphasizes the efforts of the author to minimize the difficulties that will confront the reader who wishes to acquaint himself with the chain of ideas exposed in the text.

D. J. Struik

NOTES

The Seventh American Scientific Congress, sponsored by the Department of Public Education of the Republic of Mexico, will take place September 8–17, 1935, in Mexico City. There are fourteen sections, the first being that of Physics and Mathematics. Universities and scientific organizations are invited to send representatives. Papers to be presented must be in the hands of the General Secretary by August 1. Correspondence relative to the Congress should be addressed to: Comisión Organizadora del Séptimo Congreso Científico Americano, Secretaria de Educación Pública, Mexico City, Mexico.

The annual meeting of the British Association for the Advancement of Science will be held at Norwich, September 4–11, 1935, under the presidency of Professor W. W. Watts.

The autumn meeting of the National Academy of Sciences will be held at the University of Virginia on November 18–20, 1935.

The Swiss Mathematical Society celebrated the twenty-fifth anniversary of its existence in a meeting held at the University of Berne on May 25, 1935. Professor Carathéodory, of the University of Munich, delivered an illustrated lecture on special problems and the general theory of the calculus of variations. Professor Arnold Emch, of the University of Illinois, is a charter member of this Society.

The Research Institute for Mathematics and Mechanics of the University of Tomsk has established a new journal, Bulletin de l'Institut de Mathématiques et Mécanique à l'Université Kouybycheff de Tomsk.

The Australian National Research Council has awarded its Lyle medal to Professor J. R. Wilton, Elder professor of mathematics in the University of Adelaide.

The Calcutta Mathematical Society has announced the following subject for the first competition for the Krishna Kumari-Ganesh Prasad prize and medal: Lives and works of the ten famous Hindu mathematicians: Aryabhatta, Varamihir, Bhaskara I, Lalla, Brahmagupta, Sridhar, Mahavir, Sripati, Bhaskara II, Naryana.

Commonwealth Fund Fellowships have been awarded to Mr. M. H. L. Pryce, of Trinity College, Cambridge, and to Mr. E. D. Tagg, of Claire College, Cambridge. Mr. Pryce will study at the Institute for Advanced Study and Mr. Tagg at Princeton University.

The Council of the Royal Statistical Society, London, offers the Frances Wood Memorial prize for the best investigation, on statistical lines, of any problem affecting the economic or social conditions of the people. Competition closes October 31, 1935.

The Royal University of Pisa is offering a research scholarship for the year

1935–36 under the Galileo Galilei Foundation. If the candidate is not Italian, he will be expected to carry on scientific research in an Italian institute during the tenure of the scholarship. Further information can be obtained from the Rector of the University of Pisa.

The Franklin Institute has awarded a Franklin medal to Professor Albert Einstein, of the Institute for Advanced Study, in recognition of "his contributions of fundamental importance to theoretical physics, especially his work on relativity and the photo-electric effect."

The American University at Cairo is seeking a man to teach mathematics for the year 1935–36. Board and room will be provided, but nothing is available for salary or traveling expenses to Egypt. The candidate would have to invest $500–$700 of his own money in the venture to pay expenses. Inquiries may be addressed to Mr. Hermann A. Lum, Executive Office of the American University at Cairo, 1000 Land Title Building, Philadelphia, Pa.

The American Philosophical Society has awarded grants to Professors C. E. Mendenhall and Gregory Breit, of the University of Wisconsin, in support of their experiments on nuclear disintegration and scattering with protons and deutons, and to Professor H. S. Vandiver, of the University of Texas, to enable him to continue his work on the computation and investigation of the properties of Bernoulli numbers.

The University of Wisconsin has conferred a doctorate of science on Professor G. A. Bliss, chairman of the department of mathematics at the University of Chicago and associate editor of the Annals of Mathematics.

Guggenheim Fellowships have been awarded to Professor A. H. Copeland, of the University of Michigan, and to Professor D. V. Widder, of Harvard University.

Professor Dunham Jackson, of the University of Minnesota, and Professor J. H. Van Vleck, of Harvard University, have been elected to membership in the National Academy of Sciences.

The University of Arkansas has conferred the degree of doctor of laws on Professor G. W. Mullins of Barnard College, Columbia University.

Professor D. E. Smith, of Columbia University, was nominated for election as an honorary member at the annual meeting of the British Mathematical Association held in London.

Professor Hermann Weyl, of the Institute for Advanced Study, has been elected a member of the American Philosophical Society.

Dr. Lars Ahlfors, now adjunct professor of mathematics at the University of Helsingfors, has been appointed lecturer on mathematics and tutor in the division of mathematics.

Professor Adolf Hammerstein, of the University of Berlin, has been appointed professor of mathematics at the University of Kiel.

The retirement of Professor Felix Hausdorff, of the University of Bonn, has been announced.

Professor Guido Hoheiser, of the University of Breslau, has been appointed professor of mathematics at the University of Greifswald.

Professor M. M. J. Potron, of Paris, has been appointed professor of mathematics at the Catholic Institute, Paris.

Professor H. S. Bailey, who has been a member of the staff of the department of mathematics at the Massachusetts Institute of Technology since 1891 and professor of mathematics since 1907, retired at the end of the academic year 1934–35 with the title of emeritus professor.

Dr. D. H. Ballou has been appointed assistant professor of mathematics at Georgia School of Technology for the next academic year.

Dr. J. L. Barnes, of the Massachusetts Institute of Technology, has been appointed assistant professor of mathematics at Tufts College beginning the next academic year.

Assistant Professor Samuel Beatty, of the University of Toronto, has been promoted to a professorship in mathematics.

Dr. Andrew C. Berry has been promoted to an assistant professorship in mathematics at Columbia University.

Dr. J. M. Clarkson, of Cornell University, has been appointed assistant professor of mathematics at North Carolina State College.

Professor C. W. Crockett has been named professor emeritus of mathematics and astronomy at the Rensselaer Polytechnic Institute.

Associate Professor R. F. Deimel, of Stevens Institute of Technology, has been promoted to a professorship in mechanical engineering.

Dr. B. P. Gill, of the College of the City of New York, has been promoted to an associate professorship.

Professor D. C. Gillespie has been appointed chairman of the department of mathematics of Cornell University. The retiring chairman is Professor W. B. Carver.

Dr. A. H. S. Gillson, associate professor of mathematics at McGill University, has been promoted to a professorship.

Associate Professor D. L. Holl, of Iowa State College, has been promoted to a professorship in mathematics.

Associate Professor H. I. Lane, of the University of South Dakota, has been promoted to a professorship in mathematics.

Assistant Professor W. H. McEwen, of Mount Allison University, Canada, has been promoted to a professorship in mathematics.

Assistant Professor L. C. Mathewson, of Dartmouth College, has been promoted to a professorship in mathematics.

Dr. J. E. Ostrander has retired as head of the department of mathematics of the Massachusetts State College.

Professor H. B. Phillips, of the Massachusetts Institute of Technology, has been made a member of the Visiting Committee of the physics department of Brown University.

Assistant Professor I. R. Pounder, of the University of Toronto, has been promoted to a professorship in mathematics.

Dr. G. D. Robinson, of the University of Toronto, has been promoted to an assistant professorship in mathematics.

Professor D. J. Struik, of the Massachusetts Institute of Technology, is on leave of absence this year and is in Holland.

Dr. A. W. Tucker, of Princeton University, has been promoted to an assistant professorship in mathematics.

Dr. H. L. Turrittin has been appointed adjunct professor of engineering and mathematics at the College of Mines, El Paso, Texas.

Professor H. S. White, of Vassar College, has been named Senior Lecturer in mathematics.

Dr. H. P. Wirth, of the College of the City of New York, has been promoted to an assistant professorship in mathematics.

Professor Ruth S. Wood, of Smith College, has been named professor emeritus in mathematics.

Among members of the American Mathematical Society who have recently entered the field of secondary teaching are Mr. N. G. Parke, at the Hun School, Princeton, and Dr. C. Grace Shover, at the Shipley School, Bryn Mawr.

The following appointment to an instructorship in mathematics is announced.

Cornell University: Dr. D. C. Lewis.

Ulrico Hoepli, of Milan, Italy, editor and publisher, died on January 24, 1935. He had received an honorary doctorate from the University of Zurich and had been made an honorary fellow of the Milan Academy of Fine Arts.

Professor H. M. MacDonald, professor of mathematics at the University of Aberdeen since 1904, died on May 16, 1935. He was a fellow of the Royal Society and of the Royal Astronomical Association and was president of the London Mathematical Society.

Professor Ganesh Prasad, Hardinge professor of pure mathematics at the University of Calcutta since 1923, died on March 9, 1935, at the age of eighty-one.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

264. Professor C. G. Latimer: *On the fundamental number of a rational generalized quaternion algebra.*

Let $\mathfrak{A}$ be a rational generalized quaternion algebra with a basis $1, i, j, ij$, where $i^2=-\alpha$, $j^2=-\beta$, $ij=-ji$; α and β being integers, neither divisible by the square of a prime. Brandt defined the fundamental number d of $\mathfrak{A}$ and showed that two algebras with the same d were equivalent. (*Idealtheorie in Quaternionenalgebren*, Mathematische Annalen, vol. 99 (1928), pp. 9–12.) In this paper, d is determined explicitly in terms of certain divisors of α and β. In a recent paper, Albert showed that $\mathfrak{A}$ has a basis in the above form with $\alpha=-\tau$, $\beta=-\sigma$ where τ, σ satisfy certain conditions. (*Integral domains in rational generalized quaternion algebras*, this Bulletin, vol. 40 (1934), pp. 167–168.) As an application of the present theorem, it is found that Albert's $\sigma=-d$ and that his τ may be any integer satisfying the conditions mentioned above. (Received May 15, 1935.)

265. Professor E. T. Bell: *General relations between Bernoulli, Euler, and allied polynomials.*

The relations in question form a complete set, connecting any $n(n=1, 2, 3, 4)$ of the polynomials, which appear in the relations as the arguments of arbitrary functions, the arguments being linear in the polynomials (precisely, in the umbrae of the polynomials). There are also general relations connecting polynomials whose ranks are in arithmetical progression. By setting the variable equal to zero or one in the relations, corresponding relations are obtained for the associated numbers. (Received May 18, 1935.)

266. Mr. Garrett Birkhoff: *Abstract continuous groups.* I: *Linear spaces.*

It is shown that functions between any linear spaces occurring in practice can be analyzed by the methods of the calculus. The most interesting new developments are, the description of these spaces by direct *combination* of norm functions, a related definition of the "order" of a function, a characterization of polynomials by functional equations. (Received May 18, 1935.)

267. Professor R. L. Wilder: *The strong symmetrical cut-sets of closed euclidean n-space.*

It was recently shown by W. Dancer (this Bulletin, vol. 41 (1935), p. 342), that the simple closed curve is the only strong symmetrical cut-set of the simple closed surface. It is the purpose of the present note to extend this result as follows: Let M be a strong symmetrical cut-set of the closed euclidean n-space. Then M is a generalized closed $(n-1)$-manifold (Annals of Mathematics, vol. 35 (1934), p. 878), whose Betti numbers $p^i(M)$, $0 \leqq i \leqq n-2$ are all zero. In particular, if $n=3$, M is the topological 2-sphere. (Received May 21, 1935.)

268. Professor E. P. Lane: *The neighborhood of a sextactic point on a plane curve.*

In this paper it is shown that, by suitable choice of the projective coordinate system, the equation representing an analytic plane curve in the neighborhood of a sextactic point (0, 0) on it, can be reduced to the form $y=x^2+a(x^6+x^8)+a_9x^9+a_{10}x^{10}+\cdots$, $(a \neq 0)$. This canonical expansion is then used to study the curve in the neighborhood of the sextactic point, particular attention being given to certain cubic, quartic, and quintic curves having contacts of various orders with the curve at the point. (Received May 25, 1935.)

269. Professor G. A. Baker: *The probability that the mean of a second sample will differ from the mean of a first sample by less than a certain multiple of the standard deviation of the first sample.*

It is shown that the variable $v=(x-z)/y$ where x is the mean of a first sample of n_1, z is the mean of a second sample of n_2, and y is the standard deviation of the first sample is distributed as proportional to $r^{(1-n_1)/2}$ where $r=1+[n_2v^2/(n_1+n_2)]$. The sampled population is assumed to be normal. If the two samples are combined to obtain a value of y then the corresponding distribution is $s^{-(n_1+n_2)/2}$ where $s=1+[n_1n_2v^2/(n_1+n_2)^2]$. These distributions are easily transformed into "Student's" t-distribution. They can be used to calculate "true" probable errors, to test the significance of divergence of samples, and as rejection criteria. (Received May 29, 1935.)

270. Professor G. A. Baker: *The probability that the standard deviation of a second sample will differ from the standard deviation of a first sample by less than a certain multiple of the standard deviation of the first sample.*

The distribution function of $v=(x-y)/x$ where x is the standard deviation of a first sample of n_1 and y is the standard deviation of a second sample of n_2 is shown to be proportional to $(1-v)^{(n_2-2)}\left\{[(1-v)^2n_2/n_1]+1\right\}^{(2-n_1-n_2)/2}$ if the sampled population is normal. This distribution allows one to calculate "true" probable errors which are greatly in excess of conventional probable errors of standard deviations for small samples. (Received May 29, 1935.)

271. Dr. L. S. Bosanquet: *Some arithmetic means connected with Fourier series.*

In the theory of the Cesàro summability of a series Σa_n an important rôle is played by the sequence na_n. This may be traced, to a great extent, to the known identities $\tau_n^\alpha = \alpha(s_n^{\alpha-1} - s_n^\alpha) = n(s_n^\alpha - s_{n-1}^\alpha)$, where s_n^α and τ_n^α are the Cesàro means of order α of the sequences $s_n = a_0 + a_1 + \cdots + a_n$ and na_n, respectively. The object of this paper is to investigate the Cesàro means of the sequence na_n, where $\Sigma a_n \cos nt$ is a Fourier series. In particular, necessary and sufficient conditions are obtained for the function to satisfy in order that the sequence na_n should be bounded (C). (Received June 3, 1935.)

272. Dr. L. M. Blumenthal (National Research Fellow): *The metric characterization of the n-dimensional hyperbolic space.*

In this paper the n-dimensional hyperbolic space $H_{n,r}$ of curvature $-1/r^2$ is characterized among semimetric spaces by means of relations between the distances of its points. If $p_1, p_2, \cdots, p_k$ are k points of a semimetric space, and we call the symmetric determinant $|\cosh p_i p_j / r|$, $(i, j = 1, 2, \cdots, k)$, the Lobatschefskian determinant of the k points, it is proved that a semimetric space S is congruent with a subset of $H_{n,r}$ if and only if (i) for every integer $s \leqq n+1$, the Lobatschefskian determinant of each set of s points of S has the sign $(-1)^{s-1}$ or vanishes; (ii) the Lobatschefskian determinant of each set of $n+2$ points of S vanishes; while, in case S consists of exactly $n+3$ points, we adjoin (iii) the Lobatschefskian determinant of the $n+3$ points vanishes. If we suppose that the space S is convex, complete, and of finite dimension, conditions that it be isometric with a convex subset of $H_{n,r}$, may be given in terms of Lobatschefskian determinants formed for merely each quadruple of its points, after the manner of the author's *Concerning spherical spaces* (American Journal of Mathematics, vol. 57 (1935), pp. 51–61). (Received June 4, 1935.)

273. Professor V. G. Grove: *Differential geometry of a certain type of surface in S_4.*

The purpose of this paper is to study surfaces sustaining an orthogonal conjugate net and immersed in a space of four dimensions. As is well known the normals to the surface lie in a unique normal plane. The author studies the given surface by the use of certain surfaces in three dimensions called *normal projection surfaces.* Each normal to the surface determines a unique normal projection surface. Among the normals to the given surface there are two unique normals which are perpendicular and are such that the normal projection surfaces determined by them have the one a maximum and the other a minimum total curvature. These unique principal normals permit writing the system of partial differential equations in a canonical form. Certain analogues of theorems concerning surfaces in three dimensions are given. For example, a curve is a geodesic if and only if the osculating plane of the curve at each of its points intersects the normal plane of the surface at these points in a line. If this line generates a congruence conjugate to the congruence, the geodesic is a plane curve. (Received June 7, 1935.)

274. Miss M. C. Wolf: *Symmetric functions of matrices.*

This paper modifies the fundamental theorem concerning elementary symmetric functions so that it may be applied to matrices. Since the commutative law of multiplication does not hold in general, it has been necessary to define sets of fundamental symmetric polynomials which are a direct generalization of the elementary symmetric functions, containing them as a special case when the variables are commutative. Any polynomial of degree m symmetric in the non-commutative elements $x_1, x_2, \cdots, x_n$ is equal to a polynomial, with integral coefficients, in the elements of any fundamental set and the coefficients of the original polynomial. The representation is unique. In the commutative case n elementary symmetric functions are sufficient to represent the symmetric polynomials of any degree m involving n variables. In the general case this does not hold; an additional finite set of fundamental polynomials is necessary for each higher degree. Although the choice of a fundamental set is not unique, the number for each degree is unique. A method for calculating these numbers is given. For the first five degrees they are 1, 1, 2, 6, 22. One particular simple fundamental set is defined and discussed. (Received June 8, 1935.)

275. Mr. H. J. Hamilton: *Transformations of multiple sequences.*

The double sequence $\{\sigma_{mn}\}$, where $\sigma_{mn} \equiv \sum_{k,l=1}^{\infty} a_{mnkl} s_{kl}$, is said to be the transform of the double sequence $\{s_{kl}\}$ by means of the infinite matrix $\|a_{mnkl}\|$. Definitions of ultimate boundedness, boundedness, convergence, bounded convergence, ultimately regular convergence (that is, convergence plus ultimate row and column convergence), bounded ultimately regular convergence, and regular convergence being given, it is possible to find necessary conditions on the matrix $\|a_{mnkl}\|$ in order that $\{\sigma_{mn}\}$ have a specified one of the above properties whenever $\{s_{kl}\}$ has a specified one of these properties, with existence of the transform for each m, n hypothesized; and to find sufficient conditions for the same situations, with or without existence of the transform for each m, n. In this paper such conditions are determined for the analogous transformations of n-tuple sequences in general. Several auxiliary types of sequences are introduced, and there is given a discussion of limit-preservation when $\{s_{kl}\}$ is convergent. (Received June 8, 1935.)

276. Professor E. T. Browne: *On the matric equations* $P(A, X) = 0$ *and* $P(X) = A$.

Let A be a given square matrix of order n, and let $F_i(A)$, $(i = 0, \cdots, m)$, be given polynomials in A with scalar coefficients. The purpose of this paper is to investigate the existence of square matrices X of order n satisfying the equation $P(A, X) \equiv \sum_{i=0}^{m} F_i(A) X^{m-i} = 0$. In the particular case where $F_i(A)$, $(i = 0, \cdots, m-1)$, reduce to constants p_i, and $F_m(A) = p_m - A_m$ the above equation reduces to the special case $P(X) = A$. In this paper only such matrices X as are expressible as polynomials in A are considered, and it is shown that the principal *idempotent* and *nilpotent* matrices associated with A lend themselves very readily to a simple and elegant solution of the problem. (Received June 11, 1935.)

277. Professor Leonard Carlitz: *On the representation of a polynomial in a Galois field as the sum of an odd number of squares.*

The problem is that of determining the number of sets of polynomials X_i, $(i=1, \cdots, 2s+1)$, each of degree k such that $\alpha L=\alpha_1 X_1^2+\cdots+\alpha_{2s+1}X_{2s+1}^2$, where $\alpha, \alpha_1, \cdots, \alpha_{2s+1}$ are in the Galois field, and $\alpha_1+\cdots+\alpha_{2s+1}=0$ or $\neq 0$ according as $l=$degree of $L<$ or $=2k$. In the present paper the results of an earlier communication are extended in various directions. The α_i are now quite arbitrary. In the case $l<k$, $L=L_0M^2$, where L_0 contains no quadratic factors, it is shown that the ratio of the number of representations for L to the number for L_0 is a divisor function of M. (Received June 17, 1935.)

278. Professor Leonard Carlitz: *On sums of squares of polynomials.*

This note supplements two papers on the representation of a polynomial in a Galois field as the sum of an assigned number of squares. The problem considered here is that of finding the number of solutions of $\alpha_1X_1^2+\cdots+\alpha_sX_s^2=0$ in polynomials X_i of degree k; the α_i are in the Galois field. (Received June 17, 1935.)

279. Professor Leonard Carlitz: *On certain higher congruences.*

This paper deals with the congruence $\pi(t-G)\equiv A \pmod P$: A, P, G, denote polynomials in an indeterminate x with coefficients in a fixed Galois field. The product extends over all G (zero included) of degree less than some fixed m. The congruence has either no solutions at all, or else has p^{nm} distinct solutions, where p^n is the order of the Galois field, m is less than the degree of P, and P is assumed to be irreducible. Simple criteria for the solvability of the congruence are given. (Received June 17, 1935.)

280. Dr. Deane Montgomery (National Research Fellow) and Dr. Leo Zippin: *Periodic one-parameter groups in three-space.*

In this paper a one-parameter group of homeomorphisms of euclidean three-space E into itself is considered. A certain weak periodic character with respect to the points of E is postulated for the group, and it is shown to be closely related to the rotation group of three-space, the fixed points constituting an infinite topological line which acts as the axis of a quasi-rotation. The present theorem may be formulated as follows: A continuous, one-parameter group $T(x; t)$, x in E, t any real number, of point and wise-periodic homeomorphisms of E into itself, whose period-function is bounded in every sphere, is a topological quasi-rotation group. In particular, it is the topological rotation group whenever the period-function is constant over the moving points. (Received June 13, 1935.)

281. Professors H. E. Buchanan and W. L. Duren: *On the characteristic exponents in certain types of problems of mechanics*

H. E. Buchanan published in the American Journal, vol. 45 (1923), and vol. 50 (1928), a discussion of the stability of the straight line and the equilateral triangle positions in the problem of three finite masses. He discussed in the American Mathematical Monthly, vol. 38 (1931) and vol. 40 (1933), the stability of the so-called helium atom for the straight line and equilateral triangle solutions. In all four of these problems the characteristic exponents 0, 0, $\pm i\omega$ occurred, ω being the angular velocity of rotation of the system. This paper gives the conditions under which these exponents will occur when the coordinates of the three bodies are referred to axes rotating uniformly about the Z-axis with angular velocity ω. This paper shows that if the equations of motion of the system have an integral which is such that the corresponding integral of the equations of variation is periodic in t with minimum period T, incommensurable with t in the case of generalized equilibrium, then two of the characteristic exponents are $\pm 2\pi i/T$. The differential equations of the three body problem and of the helium atom have ten integrals. The center of gravity integrals are used to eliminate six of the variables. Of the four integrals remaining, two are independent of the time and thus account for the exponents 0, 0. The other two integrals are periodic with period $2\pi/\omega$ and hence require that the exponents $\pm i\omega$ occur. (Received June 20, 1935.)

282. Professor R. P. Agnew: *Generalizations of the Riemann-Lebesgue theorem.* I.

The well known Riemann-Lebesgue theorem states that if $\phi(x) \equiv x$, $f(x) \in L$, and $F(u) = \int_{-\infty}^{\infty} f(x) e^{iu\phi(x)} dx$, then $\lim_{u \to \infty} F(u) = 0$. In this paper we show that the class of real measurable functions $\phi(x)$ having the property that for each $f \in L$ and $\varepsilon > 0$ there is a constant M such that $\int_B^{B+h} |F(u)| du < \varepsilon h + M$ for all B and all $h \geq 0$ is identical with the class of dispersed functions $\phi(x)$, i.e. real measurable functions $\phi(x)$ having the property that for each constant c the set of values of x for which $\phi(x) = c$ has measure 0. We give related results, and an application to Fourier transforms of functions of bounded variation. (Received June 21, 1935.)

283. Dr. G. de B. Robinson: *On the fundamental region of an orthogonal representation of a finite group.*

If H is a sub-group of G, then G may be represented as a permutation group on the cosets of H. Such a representation gives rise to a set of permutation matrices which constitute a reducible representation of G, an irreducible component (α) appearing with a multiplicity $S_H{}^{\alpha} = (1/h)\Sigma x_R{}^{\alpha}$ where the summation extends over those R's contained in H. In the present paper it is shown that the functions $S_H{}^{\alpha}$ have an important geometrical significance, and yield information concerning the definiteness of the fundamental region of the corresponding irreducible representation, if we suppose it to be orthogonal. These ideas lead naturally to a geometrical interpretation of the characteristic, which has played no part in the geometrical theory as originated by Klein, though it is fundamental from the algebraic point of view. (Received June 24, 1935.)

284. Dr. H. S. Grant: *A generalization of a cyclotomic formula.*

Jacobi stated without proof (Journal für Mathematik, vol. 30 (1846), p. 167) the following cyclotomic formula: $F(-1)\,F(\alpha^2)=\alpha^{2m}\,F(\alpha)\,F(-\alpha)$, where $F(\alpha)=x+\alpha x^g+\alpha^2x^{g^2}+\cdots+\alpha^{q-2}x^{g^{q-2}}$, q. is an odd prime, g is a primitive root, mod q., $g^m\equiv 2$, mod q., $x^{q}=1$ $(x\neq 1)$, $\alpha^{q-1}=1$ $(\alpha\neq 1)$. This relation reduces essentially to one connecting two Jacobi ψ-functions. By making use of the generalized Jacobi-Kummer cyclotomic function, and its prime ideal factorization (H. H. Mitchell, Transactions of this Society, vol. 17 (1916) pp. 165–177), this formula is extended to the case $q.^t\equiv 1$, mod n, q. an odd prime, and t any exponent for which the congruence holds, n even. In the above formula $t=1$, $n=q.-1$. (Received June 28, 1935.)

285. Professor A. A. Albert: *Simple algebras of degree p^e over a centrum of characteristic p.*

The author considers normal simple algebras A of degree p^e over F of characteristic p. It is assumed that F is such that not every such A is a total matric algebra, and it is proved that A is cyclic if and only if A has a maximal sub-field $F(y)$, $y^{p^e}=\nu$ in F. As a consequence, a direct product of two such cyclic algebras is always cyclic. Moreover every normal simple algebra A of degree p^e over F of characteristic p is similar to a cyclic algebra (of degree p^e). The exponent of A is proved to be p^t, where t is the least exponent of all splitting fields $F(y_1, \cdots, y_r)$, $y_ip^t=d_i$ in F, of A. Finally A has exponent p^t if and only if A is similar to a direct product of cyclic normal division algebras D_i of degrees $p^{e_i}\leqq p^t$ over F, such that D_1 has degree p^t, the exponent of each D_i being its degree. (Received July 1, 1935.)

286. Dr. S. C. Kleene: *General recursive functions of natural numbers.*

Using a definition of general recursive function due to Herbrand and Gödel, the author shows that every general recursive function is expressible in the form $\psi\{\epsilon y[\rho(x_1, \cdots, x_n, y)=0]\}$ where ψ and ρ are ordinary recursive functions (that is defined from 0 and $x+1$ by substitutions and recursions on one variable) and $(x_1, \cdots, x_n)$ (Ey) $[\rho(x_1, \cdots, x_n, y)=0]$; and conversely. The systems of equations which define recursive functions under this general definition cannot be recursively enumerated, since a recursive enumeration of them would make possible the construction of a new recursive function by diagonalizing. Likewise, no recursive process of deciding which systems define recursive functions is attainable. Since the condition that a system define a recursive function of one variable is expressible in the form (x) (Ey) $[\rho(s, y)=0]$, this gives a somewhat different proof than Gödel's that there are undecidable propositions in any mathematical logic satisfying certain conditions. For otherwise the logic could be used to decide recursively which systems of equations define recursive functions of one variable. Every problem of the form, whether (x) $(Ey)[\sigma(x, y)=0]$ holds $[\sigma(x, y)$ recursive], is included in the general problem, which systems of equations define recursive functions of one variable. There are also non-recursive functions of the form, $\tau(x)=0$ or 1 according as $(y)[\rho(x, y)=0]$ holds or does not hold. (Received July 1, 1935.)

287. Dr. S. C. Kleene: *λ-definability and recursiveness.*

Let a function of positive integers be called λ-definable if it is formally definable in the sense of the author (American Journal of Mathematics, vol. 57 (1935), p. 219). Using the general definition of recursive function due to Herbrand and Gödel (see abstract 41–7–286), the author shows that every recursive function is λ-definable, and conversely. The Gödel method of representing functions by numbers makes possible the extension of these results (under certain restrictions) to the case of functions of which the values are well-formed formulas. (Received July 1, 1935.)

288. Dr. S. S. Wilks: *The sampling theory of systems of variances, covariances, and intraclass covariances.*

By means of factoring and transforming characteristic functions of second order symmetric functions of normally distributed variables, a method is given for determining the sampling distributions of certain systems of variances, covariances, and intraclass covariances with special application to the generalized intraclass correlation problem for m "families" of k variates each. The method is applied to the problem of determining all of the independently distributed sums of squares associated with layouts of the Latin square, random block, and equalized random block types in replicated agricultural experiments. (Received July 2, 1935.)

289. Professor R. S. Burington: *On the direct sum in circuit theory.*

The concept of direct sum is introduced into the matric theory development of electric circuit theory to clarify the theory relating to the interconnection of a finite number of networks. The present treatment should meet certain objections to the use of the "connection tensor" appearing in the writings of Quade, Cauer, Kron, and others. (Received July 3, 1935.)

290. Professor A. F. Moursund: *On the Abel-Poisson summability of derived series of the conjugate Fourier series.*

This note gives theorems for the rth derived series of the conjugate Fourier series which are analogous to theorems given by B. N. Prasad (Journal de Mathématiques Pures et Appliquées, vol. 11 (1932), Theorem 3) and A. Plessner (Mitteilungen des Mathematischen Seminars der Universität Giessen, 10 (1923), Theorem 3) for the conjugate Fourier series. (Received July 5, 1935.)

291. Professor G. T. Whyburn: *Concerning rationality bases for curves.*

In a recent paper (Monatshefte für Mathematik and Physik, vol. 42 (1935), pp. 37–44) Knaster shows that in any stably regular curve (that is a curve which remains regular on the addition of any regular curve) every rationality basis is also a regularity basis; and he raises the question as to whether this property characterizes the stably regular curves among the rational curves. This question may be answered in the negative, because it is possible to

prove that *every dendrite* has this property and it is well known that not all dendrites are stably regular. However, in the present paper it is shown further that if a rational curve H enjoys this property then every true cyclic element of H is stably regular. Thus, by extending Knaster's result somewhat we obtain the following characterization: In order that a rational curve H should have the property that every rationality basis in H be a regularity basis in H it is necessary and sufficient that H be locally connected and that every true cyclic element be stably regular. (Received July 7, 1935.)

ERRATUM

Volume 41, page 331, abstract no. 200 (by Professor C. N. Moore): in the next to the last sentence, "$(N; c)$ to UV" should be replaced by "$(N; C)$ to UV, where $C_n = c_0 + c_1 + \cdots + c_n$."

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OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

ANN ARBOR, MICHIGAN, SUMMER MEETING AND COLLOQUIUM, September 10-14, 1935.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wisconsin, not later than August 6. Abstracts received by July 6 will appear in the July issue of the BULLETIN. Professor H. S. Vandiver will deliver a series of colloquium lectures on *Fermat's last theorem and related topics in number theory.* By invitation of the Program Committee, Professor G. Y. Rainich will deliver an address entitled *Product integrals*, and Professor G. T. Whyburn an address entitled *On the structure of continua.*

NEW YORK CITY, October 26, 1935.

Abstracts must be in the hands of Associate Secretary J. R. Kline, Low Memorial Library, Columbia University, New York City, not later than September 28. Abstracts received by the Secretaries by September 6 will appear in the September issue of this BULLETIN. By invitation of the Program Committee, Professor J. L. Walsh will deliver an address on *Interpolation by rational functions.*

LEXINGTON, KENTUCKY, November 29-30, 1935.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wisconsin, not later than November 1. All these will appear in the November issue of this BULLETIN. By invitation of the Program Committee, addresses will be delivered by Professor L. L. Dines on *Convex domains and linear inequalities*, and by Professor C. G. Latimer on *The arithmetic of generalized quaternions.*

CALIFORNIA INSTITUTE OF TECHNOLOGY, PASADENA, November 30, 1935.

Abstracts must be in the hands of Associate Secretary T. M. Putnam, University of California, Berkeley, California, not later than November 2. All these will appear in the November issue of this BULLETIN.

ST. LOUIS, MISSOURI, ANNUAL MEETING, December 31, 1935–January 3, 1936.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wisconsin, not later than November 26, 1935. Abstracts received by the secretaries by November 2 will appear in the November issue of this BULLETIN. By invitation of the Program Committee, addresses will be delivered by Professors Szegö and Synge.

R. G. D. RICHARDSON, *Secretary of the Society.*

Articles for insertion in the BULLETIN should be addressed to E. R. HEDRICK, Editor of the BULLETIN, University of California at Los Angeles. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

Subscriptions to the BULLETIN, orders for back numbers, and inquiries in regard to non-delivery of current numbers should be addressed to the American Mathematical Society, 450-459 Ahnaip St., Menasha, Wis., or Low Memorial Library, Columbia University, New York City.

The initiation fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, September, 1934, p. 66), are payable to the Treasurer of the Society, Professor G. W. Mullins, Low Memorial Library, Columbia University, New York City.

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